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CAMBRIDGE PROBLEMS:

BEING

A COLLECTION

OF THE

PRINTED QUESTIONS

PROPOSED TO THE

Candidates for the Degree of Backelor of Arts,

AT THE

GENERAL EXAMINATIONS,

From the year 1801 *to the year* 1810 *inclusive.*

WITH A PREFACE

BY A

Graduate of the University.

CAMBRIDGE:

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REPEATED favours demand repeated acknowledgements. The Publisher of the CAMBRIDGE PROBLEMS *eagerly seizes the present opportunity, to declare his obligations to the kindness which he has again experienced from the gentlemen whose names are prefixed to the papers which compose the following volume. By their liberal concessions, he has been enabled to present to the public, a compleat collection of all the Printed Questions that have been proposed to the candidates for the degree of Bachelor of Arts, at the General Examinations, during the last ten years.*

Respect for the labours of those gentlemen and a desire that no difficulty, which diligence might prevent, should retard the efforts of students in the university, alike required that the questions should be given with all possible accuracy. Notwithstanding the pains that have been bestowed in revising the proof sheets, some errors have been discovered. Let it be pleaded in extenuation, that, during the time in which a great part of the work was printed, the corrector of the press resided at a considerable distance from Cambridge.— Should any errors occur, beside those which have been pointed out, they will not often, it is hoped, be the cause of obscurity.

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PREFACE.

THE collection now presented to the public may, it is presumed, not unfairly lay some claim to utility. The labours of those learned persons, to whom is confided a most arduous and important charge, may, by being thus brought together, have secured to them that preservation which they surely merit: the attentive student may have ample means of exercising his ingenuity, and of acquiring a facility of investigation which may finally command success; and, while the purposes of Academical education are more immediately regarded, the general interests of science may not be totally overlooked.

These are, it will be allowed, important objects. One good effect, however, may be produced, to which circumstances, not very honourably marking the times, irresistibly direct our attention. *The public at large may be enabled to form some opinion of the nature and extent of the* MATHEMATICAL *and* PHILOSOPHICAL KNOWLEDGE *which a Candidate for University honours is expected to have attained.* Let this matter be a little farther considered.

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All thinking persons will acknowledge the mighty influence, on the most important concernments of this country, which must ever be possessed by our universities, when flourishing in well-deserved reputation. Their spreading fame will then only, by men of good principles, be contemplated with perfect satisfaction, when the conviction is firm, that their fame is merited. Such men will be anxious that the grounds of their high and prescriptive claims to public confidence should be clearly understood. If, in the opinion of some persons, it should appear that those grounds are not, in all instances, entirely unobjectionable, they will still feel that the dignity of the institutions should protect them from intemperance of abuse. They know, indeed, that the censures of ignorance are uncontrolled by decorum, as its applauses are unsanctioned by discrimination. Yet however unworthy of attention they may consider the coarse and deafening clamour of illiterate malignity, their indignation will be roused, when, amid the vulgar growl, tones may be distinguished that can only be the effect of cultivation and refinement. That indignation will be heightened into horror, when they behold abilities which might excite admiration employed in furnishing arms to the violent and undisciplined, and leading them on to the attack

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of those hallowed fortresses which have been, for ages, the safeguards of all learning.

The University of Oxford, which, amid the din of obloquy on Academical establishments with which we have been stunned, had the greatest cause of complaint, has been very ably vindicated by one of her own sons: a man of fine talents and scholar-like attainments. His elevated views, his affectionate zeal, and the deep sense, which he discovers, of the importance of the cause in which he is engaged, draw forth our warmest approbation; more particularly, when we turn, disgusted, from the illiberal malevolence and unfeeling levity of his antagonists.

Let not our calumniators exult. If *we* have been silent, it is not from apprehension. No formal reply to their mis-statements is, even now, intended. The volume thus introduced to public notice appears to contain a compleat refutation of the main objection that has been advanced against the course of study laid down in the University of Cambridge. Amid mis-representations, which many are delighted to believe, and which the generality are unable to detect, it is as pleasing as it is important to have the power of referring to an authentic document; which,

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in the opinion of all competent judges, will amply vindicate the economy of that illustrious seminary.

It has been urged, that, in the system of education established at Cambridge, "the invention finds "no exercise; the student is confined within narrow "limits: his curiosity is not roused: the spirit of "discovery is not awakened." Now, let the following collection of questions, derived from almost every department of Mathematical and Philosophical learning, be examined by any person who is conversant with such subjects: let him be informed, that the student has continually before him, the time when his knowledge will be brought to the test, and when his inventive powers will be exercised in the solution of problems, such as are there presented, with which he cannot be previously acquainted: that he is incessantly stimulated by every consideration that acts most forcibly on the mind of youth,—the dread of failure—the ambition of distinction—the prospect of consequent emolument—and, the hope of future success:—let him then say whether it is possible that, under such circumstances, the energies of genius can remain inactive.

To come yet more closely to the point. Let this

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great master of the δυναμις ευρετικη (*Caledonicè* ευρητικη! ED. REV.) from whom this objection proceeds, before he composes his next declamation against the University of Cambridge, amuse himself, some evening, with solving the questions contained in one of the papers for the higher classes in the ensuing collection. Let him take, for instance, the paper by *Mr. SOWERBY (page 78); which is particularized to avoid the obvious unpleasantness of selecting from the productions of the living. Or, if this requisition should be thought too troublesome to be acceded to, and the objector be, as report says, a Professor of Natural Philosophy, let him place that paper, for

* Μετέβη δ'ἰς ἀμείνονα χῶρον!! The University has had to lament the death of this valuable person, who sunk under a decline in the the prime of life. Many, to whom he was endeared by whatever is amiable in manners, will cherish the remembrance of him with unabating affection. It would be an easy, and, to the writer of this note, a pleasing task, to dwell upon the mildness of his virtues, the rectitude of his principles, the vigour of his faculties, and the extent of his attainments. This is not, however, the proper place for a delineation of his character.

For what has been said, no apology will be offered, even should these observations be deemed somewhat out of season. Who will venture to condemn that feeling which arrested the mind at the recollection of a much-respected name, and bade it pause, for a moment, to muse on moral and intellectual excellence prematurely lost? Who would not be mortified by the reflection that a man such as HE is known to have been, was permitted to depart without one public tribute to his memory?

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two hours, before his most favourite pupil, at the close of his studies. Let him examine his investigations and tell us the result. We will be very candid: we will take the Professor's word for the number of problems of which the solution may be effected; and his judgment for the legitimacy of the methods and the truth of the results. Let us hope that so fine an opportunity will not be neglected; and that we shall soon witness a grand display of the analytical powers of a mind instructed, of course, according to his own principles, to despise the synthesis of Newton, and to operate with the calculus of the sines. We will, in the mean time, content ourselves with predicting that, whatever calculus may be employed, the student will not have to complain that his inventive powers were left unexerted.†

It has been already observed, that a systematic reply to objections is not, on the present occasion,

† We would relieve the distress which the objector seems to feel, when describing the fancied labours of our Under-graduates, by assuring him that no "young man studying mechanics is compelled to get by "heart the whole of the heavy and verbose demonstrations contained in "Keil's Introduction." It is highly probable, however, that the objector and the present writer would differ materially as to the books which they might recommend to "a young man studying mechanics."

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to be expected. The most formidable of our adversaries has, however, been fairly met. Is it too much to presume that the attack has been somewhat more than repelled? Let the enemy look to himself.—Were it worth while to become the assailants, is it quite certain that his own entrenchments are absolutely impregnable?

A wild and unexampled hostility to established discipline is that harsh characteristic of the times which rendered it highly desirable that whatever was advanced in defence of it should consist of something more substantial than general assertions and metaphysical reasonings. While the advocate for University education can appeal to such FACTS as are presented in this book, we may rationally hope that credulity itself will not be widely misled. In what Academical institution, it may be asked, beside the University of Cambridge, are the Philosophical attainments of the students, of an order high enough to admit of discrimination, by methods such as the subsequent pages exhibit?

INVENTION—GENIUS—THE PHILOSOPHICAL SPIRIT—these rare and splendid endowments are destined for ascendancy wherever they appear: and

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wretched indeed would be the policy of that plan of instruction which should fail in vindicating to them their natural superiority. But by far the largest portion of intellect, on which education has to operate, consists of that which is capable of the reception and retention of truths fairly presented to it: which, by diligent observation, acquires moderate notions of the application of which those truths are susceptible, and, by practice, a useful faculty of accomplishing that application. The principles by which a system of truly liberal and effectual education ought to be guided, may, from these considerations, be deduced with sufficient distinctness and certainty. It will be rigorously demanded that truths important in themselves, legitimately deduced, and well-connected with each other, become, by unwearied reflection on their proofs and dependencies, as familiar with the student as the first principles of human knowledge: and opportunities will be afforded, by regular gradations of difficulty, for the employment of these truths in the investigation of new ones. There must be much that industry may secure, and something that genius may grasp after. By inflexibly requiring that knowledge should be thus combined with its application, genius will be taught the wholesome necessity of study, and industry may be stimulated to the exercise of invention.

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The truths of Natural Philosophy appear to form the best ground-work for Academical education.—The information which they convey is of incalculable importance; they are regularly derived from the simplest notions; and the application of which they are capable is boundless as the universe.

From the ardour natural to youth, it is greatly to be feared that the knowledge of facts will be deemed all that is necessary for future enquiries.—This knowledge might be obtained from the bare enunciation of propositions, or from a table of contents. A just taste in Philosophy can only be acquired by a profound and scrutinizing attention to the *methods* adopted by the great Masters of Science. It is not enough that a student has gone through a demonstration, or that he has verified every step.—He must dwell upon it, till the whole can be surveyed, as it were, at one glance. He must stay to estimate the value and tendency of the proposition, and to contemplate the situation which it holds in the general system. It is only by such cautious proceeding that his acquirements can be, at once, solid and permanent, and dignified and effective. All this is true, whether the study of Mathematical Philosophy be pursued for its own sake, or as a discipline by which

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the mind is strengthened and enlarged. In times marked by self-confidence and disdain of labour, it cannot be unseasonable to assert the necessity of regular advancement; and to assure the most undoubting genius, that he is not doomed to light, at one bound, on the top-most pinnacle of science.

To those who are unacquainted with the examinations which the Candidates for the degree of Bachelor of Arts must necessarily undergo, it may not be improper to state, that those examinations last for the space of five days: that, for the purpose of ascertaining the relative proficiency of the candidates, demonstrations of questions, involving the principles of the most important parts of pure and mixed mathematics, are expected to be compleatly drawn out: and that the papers under each year, in the following collection, constitute a very small part of the whole examination; three hours only, on each of the two first days, being appropriated to them.*

It will not be expected, by considerate persons, that *all* the ensuing propositions should be new and

* Let it be observed, that in this account of University discipline, the highest classes are chiefly referred to; and that mention of the examinations in Moral Philosophy and Metaphysics is purposely omitted.

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important. Many of them present truths of real and intrinsic value. The purpose, however, for which they were constructed would probably be thought to be sufficiently attained, when an example was afforded of the combination and application of known principles.

Great discrimination and judgment must evidently be demanded in adapting the questions to the circumstances under which their solution is expected. Considerable diversity, it should appear, is desirable, both as to the sources from which they are derived, and to the difficulty which they involve. The intentions with which they are given will be equally frustrated when the principles of solution are too obvious and simple, and when they are too remote, or too difficult of application. Such, it must at the same time be admitted, is the influence of favourite pursuits on the human mind, that it would not be easy to select an instance in which, in the unanimous opinion of the learned, the practicable exercise of ingenuity had, through an entire paper of questions, been successfully kept in view.

Haste can never be accepted as an apology for incorrectness, except when revision is impossible.

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This preface, which must now terminate, was undertaken at the request of the publisher, when the volume which it announces was on the verge of publication. Let the unavoidable rapidity, with which it was drawn up, atone, in some degree, for the many imperfections, both in sentiment and expression, which it must undoubtedly exhibit. Let it be considered as the sudden effusion of one who is more accustomed to thinking than to writing; who is not wholly unacquainted with the principles of just composition, and who does not wish to be quite inattentive to them. In full consciousness that rudeness was not designed, the hardihood of right intention has not been solicitously restrained. The motives which prompted the foregoing observations, will, it is confidently expected, be contemplated with unalloyed satisfaction, even in moments of the most serious reflection; and the manner in which they are offered will not, it is hoped, be surveyed with very troublesome discontent. To have vindicated the cause of good learning must always afford a subject for exultation; and bad indeed must be that method of procedure, which can overbalance all the merit arising from the exposure of misrepresentation.

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CAMBRIDGE PROBLEMS.
1801.

Morning Problems.—Mr. Hornbuckle.

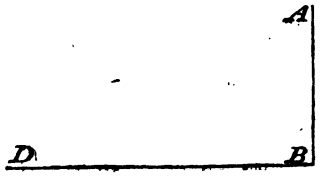
FIRST AND SECOND CLASSES.

1. COMPARE the velocities acquired in falling freely from different altitudes towards *different* centers of force, the law of force being the inverse square of the distance.

2. Investigate the equation to the reciprocal spiral, and thence determine the law of the force by which a body may describe the curve.

3. Let a given parabola be just immersed vertically in a fluid; at what distance from the vertex must a line be drawn parallel to its base, that the pressure on the upper part may be to that on the lower in the proportion of m to n.

4. A body falls freely by the force of gravity down AB, and uniformly describes the space BD, equal to twice AB, on the horizontal plane, with the velocity acquired. Determine, geometrically, the length and inclination of a plane drawn from A to BD, the time down which may be equal to the time down AB and along BD.



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5. Divide a given angle into two angles, whose tangents shall be to each other in a given ratio.

6. On a given day in a given latitude the length of the shadow cast by a tower at 12 o'clock was *(a)* feet. Required the height of the tower.

7. Find the length of an arc of the meridian corresponding to any given latitude, according to Mercator's projection, and reconcile it to the construction given by Cotes.

8. Where must an eye be placed that an object, seen through a double convex lens, may appear of the same magnitude at all distances from the lens.

9. A body is projected from the top of a given inclined plane. Required the direction of projection in which the least velocity will bring it to the bottom of the plane. Required also this velocity.

10. Compare the values of the respective infinite series $1+2x+3x^2+4x^3+$ &c. and $1-2x+3x^2-4x^3+$ &c.

11. Compare the chances of throwing an ace in two trials with one die, and in one trial with two.

12. Transform the cubic $x^3-px^2+qx-r=0$, whose roots are a, b, c, into one whose roots are $\frac{1}{a+b}$, $\frac{1}{a+c}$, $\frac{1}{b+c}$.

13. Any two sides of a spherical triangle remaining constant, determine the ratio of the nascent increments of the angle included between those sides, and of either of the other angles.

14. Suppose a given sphere to be projected in a medium whose density is $\frac{1}{3}$ that of itself; compare the velocity of projection with that remaining in the

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sphere after describing a space equal to eight diameters, the resistance to the motion arising only from the inertia of the medium.

Third and Fourth Classes.—Mr. Hornbuckle.

1. Required the price of paving a floor, whose length is 10yds. 2ft. and breath 5yds. 1ft. at 2s. per square foot.

2. Reduce 17*s*. 9½*d*. to the decimal of a pound.

3. Prove the rule for finding the greatest common measure of two quantities, and reduce $\frac{216}{3186}$ to its lowest terms.

4. If an angle of a triangle be bisected by a line which also cuts the base, the rectangle under the sides of the triangle is equal to the rectangle under the segments of the base, augmented by the square of the bisecting line.

5. Given the three angles of a triangle and the radius of the circumscribing circle, to find the sides.

6. The sum of the tangents of two arcs : diff. of tangents :: the sine of the sum of those arcs : sine of the difference.

7. Prove that a body cannot easily be balanced on a point under the center of gravity.

8. Given the length of a pendulum *(l)* that oscillates seconds, find by means of it how far a body will fall freely by the force of gravity in *t″*.

9. Given the velocity and direction of projection, find the range on a plane of given elevation, and the greatest altitude.

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10. Find the fluxions of the following quantities $\frac{x}{\overline{a^2-x^2}^{\frac{1}{2}}}$, $\frac{\sqrt{a^2+x^2}}{\sqrt{a^2-x^2}}$, a^x, a being constant.

11. Inscribe the greatest cone in a given sphere.

12. Find the chords of curvature perpendicular and parallel to the axis at a given point of the common parabola.

13. The roots of the cubic $x^3-px^2+qx-r=0$ are in musical progression, shew how they may be found.

14. Find generally the sums of the powers of the roots of an equation.

15. Explain the principle of the hydrometer, and shew that if it be made to sink to the same depth in different fluids, the specific gravities of these fluids are as the weight of the instrument in the several cases.

16. Find the center of a Meniscus, and prove it to be a fixed point.

17. Place an object before a double convex lens, so that the image may be double of the object, and erect.

18. Given the right ascension and declination of a star. Required its latitude and longitude.

19. Supposing the earth's orbit a parabola, find the apparent path of aberration of a fixed star parallel to the ecliptic.

20. The velocity at any point of a conic section : velocity in a circle at the same distance :: $\sqrt{\frac{1}{2}L \times SP}$: SY; Prove this, and shew it to be the same ratio as that of $\sqrt{HP}$: $\sqrt{AC}$, AC being the semi-axis major.

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21. How far must a body fall internally to acquire the velocity in a parabola, the force varying inversely as the square of the distance?

22. Compare, by means of a circle, the times of describing freely different spaces from the same distance towards the same center of force, the law of the force being the inverse square of the distance.

23. Explain the principle of Cassegrain's telescope, and find its magnifying power.

24. Sum the following series $1.2.4+2.3.5+3.4.6+$ &c. to n terms.

Also $\frac{1}{1.3}-\frac{1}{2.4}+\frac{1}{3.5}-\frac{1}{4.6}+$ &c. ad inf.

Fifth and Sixth Classes.—Mr. Hornbuckle.

1. $17\frac{1}{2}$ ells of cloth, each containing 5qrs. cost £6. 17. $10\frac{1}{4}$; how much will 18 yards cost at the same rate.

2. Required the interest due on £115. for $5\frac{1}{2}$ years at $4\frac{1}{2}$ per cent. per ann. simple interest.

3. Solve the following equations

$$\sqrt{x}-4=\frac{259-10x}{4+\sqrt{x}}.$$

$$\frac{10}{x}-\frac{14-2x}{x^2}=2\frac{4}{9}.$$

4. In any plane oblique angled triangle, given two sides and the included angle, to solve the triangle.

5. A body falls from rest by the force of gravity down a given inclined plane; compare the times of describing the first and last halves of it.

6. Prove the velocity in any point of a parabola equal to that acquired in falling down ¼ the parameter.

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7. Given a rectilinear object, and its inclination to a known refracting surface; required the magnitude and inclination of the image.

8. Prove that the image of a strait line before a spherical reflector is the arc of a conic section.

9. Find the time of emptying a cylinder of given base and altitude through a small orifice in its base; and investigate the fluxional expression for the time of emptying vessels in general.

10. If all the coefficients of an equation be whole numbers, shew that it cannot have a fractional root.

11. Draw a meridian line on a horizontal plane.

12. Compare the velocity in a curve with that in a circle, at the same distance, in general, and in the conic sections; the center of force being in the focus.

Evening Problems.—Mr. Walker.

1. Find the exact value of .1666, &c. of a £.

2. Find the length of a pendulum that would oscillate seconds at the dist. of the earth's rad. above its surface; and then determine the point below the surf. where it would oscillate in the same time.

3. A body, projected from the top of a tower at an angle of 45° above the horizontal direction, fell in 5″ at a dist. from the bottom of the tower equal to its altitude: find that alt. in feet.

4. Compare the resist. upon the curve of a semicircle, moving in a fluid in the direction of its axis, with the resist. upon its base.

5. Two equal hollow cubes are immersed in water, having a side parallel to the surface, and their depths are in the ratio of 4 : 1; compare the times of filling through equal orifices in the bottoms.

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6. What must be the form of a glass lens placed in water, that all rays, incident parallel to its axis, may converge accurately to one point within the water?

7. The reflecting curve is a semi-circle, and rays fall parallel to its axis: construct the caustic; and compare the density of the rays at different points.

8. Given the sun's alt. at 6 o'clock, and the alt. when due east; find the lat. of the place.

9. Given the *mean* horary motion of a planet in its orbit; shew the method of finding the horary motion in longitude.

10. The hyp. logarithm being given, find the corresponding number.

11. Sum the series $\frac{2}{1.3}\times\frac{1}{3}+\frac{3}{3.5}\times\frac{1}{3^2}+\frac{4}{5.7}\times\frac{1}{3^3}+$ &c. ad inf.

And $1.2.4+3.4.6+5.6.8+7.8.10+$ &c. to n terms, by increments.

12. Shew the method of finding the content of a pyramid, whatever be the figure of its base; the area of the base, and alt. of the solid, being given.

13. Investigate the ratio of the centripetal and centrifugal forces in any curve; and apply it to the hyperbolic spiral.

14. Find the actual vel^{y}. and per. time, of a body revolving in a circle, at the dist. of the earth's rad. above its surface.

15. The equation to a certain curve is $y=\frac{4x^2}{\sqrt{a^2-x^2}}$, where x is the absc. and y the ord.: find the whole area of the curve, and shew that it = area of a circle whose rad. is a.

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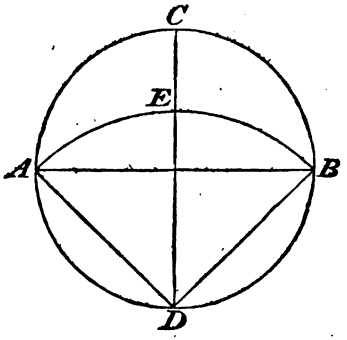
16. Shew the method of comparing the mean addititious force of the sun upon the moon, with the force of gravity at the earth's surface.

17. Let AB and DC be two diam. of a given circle, drawn at right angles to each other; AEB a circular arc described with rad. DB or DA: prove that the area of the lune $AEBC$ = area of triangle ADB.



18. Suppose a comet, in its descent towards the sun, to impel the earth from a circular orbit, in a direction making any acute angle with the earth's dist. from the sun; and the velᵛ. after impact : velᵛ. before :: $\sqrt{3} : \sqrt{2}$: find what change would be produced in the length of the year.

19. A globe and its circumscribed cylinder revolve uniformly round a common axis in the same time: compare the motion of the cylinder with that of the globe.

20. Shew that the fluent of $\frac{\dot{x}}{x}\int\frac{\dot{x}}{1+x}$, generated whilst x from 0 becomes 1, is $=\frac{2^2}{3}$, where 2 is the quad. arc of a circle whose rad. is 1.

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21. A body descends in a parabola in a resisting medium, by a force tending to the focus, which $\propto \frac{1}{\text{dist.}}$: compare the resist. with the centrip. force: and find the law of the velocity.

22. The relation between the abscissa and ordinate of an algebraic curve is expressed by the equation $y^n - \overline{a+bx}.y^{n-1} + \overline{c+dx+ex^2}.y^{n-2}$—, &c. $= 0$: prove that the sum of the ordinates divided by the respective subtangents, is a constant quantity.

23. A cylinder of a given weight and dimensions, is put in motion round an axis parallel to the horizon, by a given weight (P) suspended by a small string wound round the surf. of the cylinder: find the actual time in which P would descend from the surf. of the earth to the center.

24. Let A and B be two particles of matter, connected by an inflexible line AB; and let a force be impressed perpen. at any point D which is not the center of gravity: find the point of initial spontaneous rotation; and then determine the path of that point in one revol. of A and B.

Morning Problems.—Mr. Walker.

FIRST AND SECOND CLASSES.

1. An equiangular prism is placed upon an inclined plane with its axis parallel to the horizon, and is just supported: — find the plane's inclination.

2. A cylinder is just immersed in a fluid with its axis perpendicular to the surface; find at what point it must be cut by a plane perpendicular to its axis,

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that the pressures upon the convex surfaces may be equal.

3. Prove that a small rectilinear object and image subtend equal angles at the vertex of a spherical reflector.

4. The latitude being given, find at what time on the longest day, the variation of the sun's altitude is greatest.

5. Given the moon's horizontal parallax and periodic time, with the sun's apparent $\frac{1}{2}$ diameter and length of a sidereal year; shew the method of comparing the densities of the earth and sun.

6. $a\dot{z}=\dot{y}\sqrt{a^2+4y^2}$, where z is the arc of a certain curve, and y the ordinate; determine the relation between the ordinate and abscissa.

7. Prove that the solid generated by the revolution of an ellipse round the minor axis, is a mean proportional between the solid generated round the major axis and its circumscribed sphere.

8. Find the distance of the point of suspension from the center of gravity of a given system of bodies, that the time of an oscillation may be the least possible.

9. Investigate and construct Cotes's fifth spiral.

10. The moon performs a revolution round the earth in a certain period, whilst they revolve round their common center of gravity; in what ratio must the mean distance of the moon be diminished that it may revolve round the earth at *rest*, in the same time?

11. Describe the curve which is the locus of the equation $x^4-a^2x^2+a^2y^2=0$: and find its whole area.

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12. Find the fluent of $\frac{\dot{x}}{\sqrt{1+x^2}}$, having given the fluent of $\frac{\dot{x}}{1+x^2}$: find also the fluent of $v^2x^3\dot{x}$, where v=hyp. log. x.

13. Prove the differential method of summing series, by the method of increments.

14. Find the attraction of a corpuscle to a sphere, when the attractive force to each particle $\propto \frac{1}{\sqrt{\text{dist.}}}$.

Third and Fourth Classes.—Mr. Walker.

1. Find the interest of £555. for $2\frac{1}{2}$ years, at $4\frac{1}{4}$ per cent.

2. A piece of ground, containing 4970.25 square yards, is to be laid out in the form of a square: find the length of a side.

3. Given $x+\sqrt{a^2+x^2}=\frac{2a^2}{\sqrt{x^2+a^2}}$: find x.

4. Prove that the ratio $\sqrt[m]{1+x}:1$ is nearly equal to the ratio $1+\frac{x}{m}:1$, where x is very small when compared with 1.

5. Shew the method of ascertaining the height of a mountain above the level of an horizontal plane contiguous to a sloping side.

6. Prove that the minor axis of an ellipse is a mean proportional between the major axis, and the latus rectum.

7. In a right angled spherical triangle, sine of hypoth. : rad. : : sine of a side : sine of the angle opposite that side. Required a demonstration.

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8. Given the two apparent weights in each end of a pair of false scales, find the true weight: and shew whether it is greater or less than $\frac{1}{2}$ their sum.

9. Find the height to which a body will rise, if projected perpendicularly from the horizon, with a velocity of 144 feet in 1″ : and find how far it will ascend in 2″.

10. Compare the time of an oscillation in a cycloid, with the time in which a body would fall through a space = length of the pendulum.

11. A cubical vessel, whose altitude is 32 inches, stands upon an horizontal plane and is filled with water: — find where a small orifice must be made in a side, that the fluid may spout to a distance equal to the height of the vessel.

12. Find the height of an homogeneous atmosphere: and shew that it is about $5\frac{1}{2}$ miles.

13. Find the principal focus of rays refracted through a sphere, denser than the ambient medium: and, supposing the focus to be in the surface of emergence, determine the ratio between the sines of incidence and refraction.

14. Determine the place of a double convex lens, between the eye of a spectator and an object at a given distance, that the apparent magnitude may be a maximum.

15. Construct two angles *geometrically*, whose sines are in the ratio a : b, and tangents in the ratio m : 1; and prove that rad. : cos. of greater ang. : : $\sqrt{m^2-1}.b^2 : \sqrt{a^2-b^2}$.

16. Prove that $\dot{x} = \frac{r^2 \dot{s}}{s\sqrt{s^2-r^2}}$, where r and s are rad. and secant of the arc. x.

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17. Find the fluent of $\frac{z^{\frac{1}{2}n-1}\dot{z}}{\sqrt{a^n+z^n}}$; and of $v^2\dot{x}$ where $v=$ hyp. log. x.

18. Sum the series $\frac{1}{1.3}+\frac{1}{2.4}+\frac{1}{3.5}+$, &c. ad infinitum. And $1^2+2^2+3^2+$, &c. ... to n terms.

19. Construct an horizontal dial for a given latitude; and find the angle between the hour lines of 12 and 3.

20. Given the sun's declination and lat. of the place; find the time of its rising.

21. Shew that every equation, whose roots are possible, has as many changes of the signs as it has positive roots.

22. Prove that the periodic times in all ellipses round the same center are equal: and, round different centers, that they $\propto \frac{1}{\sqrt{\text{abs. force}}}$

23. Investigate the ratio of the angular velocities of the distance and perpendicular upon the tangent, in any curve: and apply it to the log. spiral.

24. A body descends from a given altitude by a force which $\propto \frac{1}{\text{dist.}^2}$, and, at the middle point of its descent, is projected with the velocity acquired, in a direction making an acute angle with the distance from the center: —what orbit will the body describe? and what will be its periodic time, when compared with the periodic time in a circle whose rad. is the given alt.?

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Evening Problems.—Mr. Hornbuckle.

1. Investigate the rule for extracting the cube root, and apply it to find the cube root of 738,763264.

2. Required the discount on a given sum *(£p)* due $1\frac{1}{2}$ years hence, at 5 per cent. per ann.

3. Resolve the recurring series $a+bx+cx^2+dx^3+$ &c. whose scale of relation is $f+g$, into two geometrical series.

4. Two equal weights are suspended by a string passing over three tacks, which form an isosceles triangle, the base being parallel to the horizon, and the vertical angle 120°. Compare the respective pressures on the tacks with each other, and with the weights.

5. Suppose a strait rod to be partly immersed in a vessel of water; determine the angle at which it must be inclined to the surface, that the apparent bending at the surface may be a maximum.

6. A cylinder full of water, whose length is equal to the diameter of the base, is supported with its sides parallel to the horizon: compare the time of discharging half the fluid through a small orifice in the lower side in this situation, with that of discharging the same quantity through an equal orifice in the base, when the sides are perpendicular to the horizon.

7. Apply Napier's rule to find the declination of a star, which, in a given latitude, rises in the north east point.

8. Investigate the nature of the curve in which a body descends from one given point to another in the least time possible; the velocity at each point

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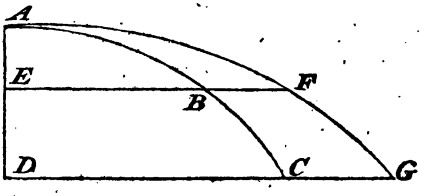
being supposed to vary as the corresponding ordinate of the curve.

9. A cylindrical rod suspended at one end, whose weight is (W) and length (l) inches, oscillates seconds; on what part of the rod must a given weight (w) be suspended that it may oscillate twice in a second.

10. Approximate to the roots of the equation $x^2+xy=5$, $2xy-y^2=2$, and shew on what the accuracy of an approximation depends.

11. Find the area of the curve, whose equation is $y=a\times$ hyp. log. $\frac{a-\sqrt{a^2-x^2}}{a+\sqrt{a^2-x^2}}$ contained between the values (a) and (b) of the abscissa; x being the abscissa and y the ordinate.

12. ABC is a semi-cycloid, DC its base, AD its axis, AFG a curve traced out by assuming the ordinate EF always equal to the cycloidal arc AB: investigate the equation to the curve AFG, and compare its area with the area of the cycloid.



13. Sum the following series $1.3^2+3.5^2+5.7^2+$ &c. to n terms

$$\frac{1}{1.2}-\frac{1}{2.4}+\frac{1}{3.6}-\frac{1}{4.8}+\text{ \&c. ad inf.}$$

$$\frac{1}{1.2.4}-\frac{1}{2.3.5}+\frac{1}{3.4.6}-\text{ \&c. ad inf.}$$

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14. The roots of the equation $x^n - px^{n-1} + qx^{n-2}$ — &c. $= 0$ are in arithmetical progression; the least root is $\frac{p}{n} - \frac{n-1}{n} \cdot \sqrt{\frac{n-1 \,.\, 3p^2 - 6nq}{n^2-1}}$, and the common difference $\frac{2}{n} \cdot \sqrt{\frac{n-1 \,.\, 3p^2 - 6nq}{n^2-1}}$. The investigation is required.

15. Find the fluents of the following quantities

$$\frac{\dot{x}}{x^3 \times \overline{a+x}^{\frac{1}{2}}} \cdot \frac{b\dot{y}}{\overline{1-y}^{\frac{1}{2}} \,.\, \sqrt{1+y^2}} \cdot \frac{a\dot{x}}{x\sqrt{a^2 - ax + x^2}}$$

16. Prove that when the first point of Aries rises, the ecliptic makes the least angle with the horizon, and when it sets, the greatest; and thence explain the phænomenon of the harvest moon.

17. Suppose a body describing a logarithmic spiral in a resisting medium, the density varying inversely as the distance, and the centripetal force as the square of the density, to be deprived of its angular motion at a given distance from the center: compare the time of its descent to the center in a strait line, with the time of descent in the spiral.

18. Two bodies, whose weights are A and B, are projected together with the respective velocities, a and b, from the same point, in the same direction, and at a given angle of inclination to the horizon. Required the greatest altitude to which their common center of gravity will ascend, and the path described by it.

19. A cylindrical vessel, of given altitude and base, is situated on a horizontal plane; an eye is placed so as to see only the farther extremity of that diameter of the base which passes through the point,

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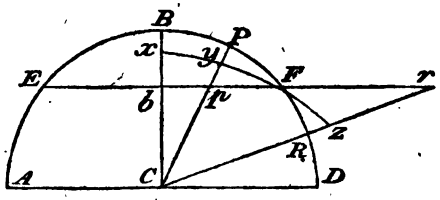
in which a perpendicular drawn from the eye to the plane meets it: To what depth must it be filled with water, that the eye in the same situation may see the center of the base?

20. Construct a vertical south-east dial.

21. ABD is a semi-circle, AD its diameter, EF any chord parallel to AD, produced indefinitely, CB, CP, CR radii; the parts Bb, Pp, Rr of the radii, intercepted between the chord, or the chord produced, and the circumference, are bisected in x, y, z: find the nature of the curve passing through all the points of bisection, both within, and without the circle.



22. Investigate the number expressing the probability of throwing an ace at least (t) times in (n) trials with a die of (p) faces marked 1, 2, 3 ... p.

23. Shew that Newton has properly applied the principles of the golden rule in his investigation of the ratio between the equatorial and polar diameters of the earth; i. e. if this ratio be that of $1+n$: 1, n being very small, and the figure of the earth an oblate spheroid, prove that the excess of weight supported at the equator is proportional to the difference (n) of the diameters.

24. Find at what angle a plane, which is perpendicular to the plane of the meridian, must be

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inclined to the horizon of a given place, that the diurnal path of the shadow of the extremity of an object, erected perpendicular to the horizon, may be a parabola on the plane: the sun's declination being given, and supposed invariable during the course of one day.

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Morning Problems.—Mr. Palmer.

FIRST AND SECOND CLASSES.

1. Given the three angles of a plane triangle, and the radius of its inscribed circle, to determine its sides.

2. The specific gravities of two fluids, which will not mix, are to each other as n: 1, compare the quantities which must be poured into a cylindrical tube, whose length is a *(a)* inches, that the pressures on the concave surfaces of the tube, which are in contact with the fluids, may be equal.

3. Determine that point in the arc of a quadrant from which two lines being drawn, one to the center and the other bisecting the radius, the included angle shall be the greatest possible.

4. Required the linear aperture of a concave spherical reflector of glass, that the brightness of the sun's image may be the same when viewed in the reflector and in a given glass lens of the same radius.

5. Determine the evolute to the logarithmic spiral.

6. Prove that the periodic times in all ellipses about the same center are equal.

7. The distance of a small rectilinear object from the eye being given, compare its apparent magnitude when viewed through a cylindrical body of water with that perceived by the naked eye.

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8. Find the fluents of the quantities $\frac{d\dot{x}}{x.\overline{a^2-x^2}}$, and $\frac{h\dot{y}}{y.\overline{a+y}^{\frac{1}{2}}}$.

9. Through what space must a body fall internally, towards the center of an ellipse, to acquire the velocity in the curve?

10. Find the principal focus of a globule of water placed in air.

11. Determine, after Newton's manner, the law of the force acting perpendicular to the base, by which a body may describe a common cycloid.

12. Find the area of the curve whose equation is $xy=a^x$.

13. What is the value of the quantity q so that force $\times \overline{\text{period}}^2 = q \times$ radius of circle.

14. Two places, A and B, are so situated that when the sun is in the northern tropic it rises an hour sooner at A than at B; and when the sun is in the southern tropic it rises an hour later at A than at B. Required the latitudes of the places.

15. From what point in the periphery of an ellipse may an elastic body be so projected as to return to the same point, after three successive reflections at the curve, having in its course described a parallelogram.

MISCELLANEOUS QUESTIONS.

Trisect a right angle.

Find the diameter and diagonal of a cube whose side = 7.

Similar triangles are to each other in the duplicate ratio of their homologous sides.—Prove it.

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Prove the truth of the Arithmetical rules for the addition, subtraction, multiplication, and division of fractions.

Investigate the rule for the extraction of the square root.

Which is greatest, the ratio $\sqrt[3]{3} : \sqrt{2}$, or $\sqrt{3} : \sqrt[3]{5}$

Shew that the magnitude of an object to the eye varies inversely as the distance.

Find *(n)* arithmetic, and *(p)* geometric means between *(a)* and *(b.)*

Shew the method of measuring the circumference of the earth, and how to draw a meridian line.

Let a solid whose specific gravity = 4, float in a fluid whose specific gravity = 10. Required the proportion between the parts immersed and extant.

Divide $x^{\frac{3}{4}}$, by $x^{\frac{1}{2}} - a^{\frac{1}{2}}$ and r^n by $a^{\frac{m}{n}} - r^{\frac{m}{n}}$, to five terms in the series.

Divide ,48624097 by 179; and 14 by ,7854.

Required the value of ,583 of a pound.

Quere the reason for pointing every 2d figure in extracting the square root, and every 3d in the cube root.

Assign the physical cause of the blue appearance of the sky on a clear day, and its western redness at sun-set.

Why are natural days unequal?

From whence does it appear that the systems of Ptolemy and Tycho are false?

Given the periodic times of the seven primary planets, to find the time between their all being in conjunction together.

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Demonstrate that every angle $\propto \frac{\text{Arc.}}{\text{Radius.}}$

Prove, that in any lever, $P : W$ :: perpendicular from the center of motion, to the direction in which the weight acts : do. in which the power acts; and that when P and W act parallel, the lever will rest in any position, except the center of gravity be below the point of suspension.

Clear the equation $x^3 - \frac{px^2}{m} + \frac{qx}{n} - r = o$ of fractions.

In how many years would (a) pounds amount to (p) pounds at (r) rate of compound interest.

Find the sum of the circulating decimal ,99999.

Find the fluent of $\frac{\dot{t}}{1-t^2}$

Given the altitude of the mercury in the barometer at the top and bottom of a mountain, to find its height.

Shew, that in a plane reflector, a man will see his whole length, if the reflector be $= \frac{1}{2}$ his length.

Find the focus of the reflected rays of a concave surface.

Find the sum of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} +$ &c.; and ten terms of the series $3 + 7 + 11$ &c.

Shew that a ray of light issuing from a point and reflected to the eye describes the shortest path possible, when the angle of incidence = the angle of reflection.

Let a body be supported upon an inclined plane, first by a power parallel to the length, then by a power parallel to the base. Compare the pressure.

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A body rests upon a string between two pullies in an horizontal position, being balanced by two equal bodies, hanging on opposite sides. Find the distance of the body from the horizontal line.

Prove the Binomial Theorem, by the method of increments.

Extract the 4th root of $16a^4 - 96a^3b + 216a^2b^2 - 216ab^3 + 81b^4$.

Reduce the fraction $\frac{1}{a^{\frac{11}{4}}}$ to one whose denominator shall be a^3.

Investigate the rule for extracting the square root of any binomial surd.

Investigate the general expressions for summing an arithmetic and geometric series.

Find the value of x in the following equations:

$\frac{1}{2}x - \frac{1}{3}\sqrt{x} = 22\frac{1}{6}$; $ax^n - b\sqrt[n]{x^2} - c = -d$; $2x^4 - x^2 + 104 = 600$.

Prove that the spaces described by a body in successive times are in arithmetic progression.

Find the value of x in the following simple equations:

$x + \sqrt{a^2 + x^2} = \frac{2a^2}{\sqrt{a^2 + x^2}}$; $\sqrt[3]{x^3 - a^3} = x - a$; $\sqrt{a^2 + x^2} = \sqrt[4]{b^4 + x^4}$; and $x = \sqrt{a^2 + x \times \sqrt{b^2 + x^2}} - a$.

Explain the construction of Briggs's Tables of Logarithms.

Shew that the time of an oscillation of a body in a cycloid, is to the time of a body in the chords of a circle whose radius is equal the length of the pendulum :: circumference of a circle : 4 times its diameter.

Given a beam, and the weight that will break it,

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to find the length of a similar beam, which being similarly situated, will break by its own weight.

Find the area and length of the common parabola.

Find the fluxion of x^x.

Extract the cube root of $7+5\sqrt{2}$ nearly, and investigate the rule.

Find the fluent of $\overline{a+cz^n}^m \times dz^{3n-1}$.

Afternoon Problems.—Mr. Dealtry.

THIRD AND FOURTH CLASSES.

1. Inscribe the greatest cylinder in a given sphere.

2. Rays, which pass through a globe at equal distances from the center, are turned equally out of their course.—Required a proof.

3. Given the declination of the sun and the latitude of the place, to find the duration of twilight.

4. A cylindrical vessel, 16 feet high, empties itself in four hours by a hole in the bottom.—What space does the surface describe in each hour?

5. Prove that if two circles touch each other externally, and parallel diameters be drawn, the straight lines, which join the opposite extremities of these diameters, will pass through the point of contact.

6. A ball, whose elasticity : perfect elasticity :: n : 1, falls from a given height upon a hard plane, and rebounds continually till its whole motion is lost. —Find the space passed over.

7. If a body revolves in any curve, compare the angular velocity of the perpendicular with that of the distance.

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8. How far must a body fall externally to acquire the velocity in a circle, the force varying as the distance.

9. Given the right ascensions and declinations of two stars, to find their distance.

10. Find the velocity, with which air rushes into an exhausted receiver.

11. Let the roots of the equation $x^3 - px^2 + qx - r = 0$ be a, b and c, to transform it into another, whose roots are a^2, b^2, c^2.

12. Find the fl. $\frac{z\dot{z}}{1+2az+z^2}$ where a is less than 1; and of $\frac{x\dot{x}}{\sqrt{a^2+x^2}}$.

13. Find that point in the ellipse, where the velocity is a geometric mean between the greatest and least velocities, the force varying $\frac{1}{D^2}$.

14. Determine the position of a line drawn from a given point to a given inclined plane, through which the body will fall in the same time as through the given plane.

15. The equation $x^3 - 5x^2 + 8x - 4 = 0$ has two equal roots.—Find them.

16. Find the sum of the cube numbers $1 + 8 + 27 +$ &c. by the differential method; and sum the following series by the method of increments:

$1.2 + 2.3 + 3.4 +$ &c. n terms.

$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4}$ &c. n terms and ad infinitum.

17. If half of the earth were taken off by the impulse of a comet, what change would be produced in the moon's orbit?

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18. Prove that if the eye be placed in the principal focus of a lens, the image of a given object would always appear the same.

19. Find the time of emptying a given paraboloid by a hole made in the vertex.

20. Find the proportion between the centripetal and centrifugal forces in a curve; and apply the expression to the reciprocal spiral.

Afternoon Problems.—Mr. Dealtry.

FIFTH AND SIXTH CLASSES.

1. Prove that an arithmetic mean is greater than a geometric.

2. Every section of a sphere is a circle.—Required a proof.

3. If $\frac{1}{4}$ of an ell of Holland cost $\frac{1}{4}$£. what will $12\frac{2}{3}$ ells cost?

4. Prove the method of compleating the square in a quadratic equation.

5. Take away the second term of the equation $x^2-12x+5=0$.

6. Inscribe the greatest rectangle in a given circle.

7. Sum the following series:

$1+3+5+7+$ &c. to n terms.

$3-1+\frac{1}{3}-\frac{1}{9}+$ &c. ad inf.

$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}$ &c. ad inf.

8. Find the value of x in the following equations:

$$\frac{42x}{x-2}=\frac{35x}{x-3}$$

$$\frac{x+1}{2}+\frac{x+2}{3}=16-\frac{x+3}{4}$$

$$3x^2-14x+15=0.$$

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9. In a given circle to inscribe an equilateral triangle.

10. Two equal bodies move at the same instant from the same extremity of the diameter of a circle with equal velocities in opposite semi-circles. Required the path described by the center of gravity; find the path also when the bodies are unequal.

11. Through what chord of a circle must a body fall to acquire half the velocity gained by falling through the diameter.

12. Given the latitude of the place and the sun's meridian altitude, to find the declination.

13. Given the sun's altitude and azimuth and the latitude of the place, to find the declination and the hour of the day.

14. Prove that the vel. in a parabola : vel. in a circle at the same distance : : $\sqrt{2}$: 1 .

15. How far must a body fall internally to acquire the vel. in a circle, the force varying $\frac{1}{D^2}$?

Evening Problems.—Mr. Dealtry.

FIRST, SECOND, THIRD, AND FOURTH CLASSES.

1. Find four geometric means between 1 and 32, and three arithmetic means between 1 and 11.

2. Suppose a straight lever has some weight, and at one end a weight is suspended equal to that of the lever; where must the fulcrum be placed, that there may be an equilibrium?

3. Determine the latitude of the place, where the sun's meridian altitude is 73°. 24′. 13″. its declination south being 16°. 36′. 47″.

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4. If 2 represent the length of a quadrant, whose radius is R, and the force vary $\frac{1}{D^3}$, the time of descent half way to the center of force: the time through the remaining half :: $2+R$: : $2-R$. Required a proof.

5. P and W represent two weights hung over a fixt pulley; supposing P to descend, what space will it describe in t'', the inertia of the pulley being taken into the account?

6. If a pendulum, whose length is 40 inches would oscillate in $1''$ at the pole of a sphere, the radius of which is 4000 miles; what must be the time of rotation round its axis, that the same pendulum at the equator may oscillate twice in $3''$?

7. A given cone is immersed in water with its vertex downward; what part of the axis will be immersed, if the specific gravity of the fluid : that of the cone :: 8 : 1?

8. The axis of a wheel and axle is placed in a horizontal position, and a weight y, which is applied to the circumference of the axle is raised by the application of a given moving force p applied to the circumference of the wheel; given the radii of the wheel and axle, it is required to assign the quantity y, when the moment generated in it in a given time is a maximum, the inertia of the wheel and axle not being considered.

9. Would Venus ever appear retrograde according to the Tychonic system?

10. A perfectly elastic ball begins to fall from a given distance SA in a right line towards the center

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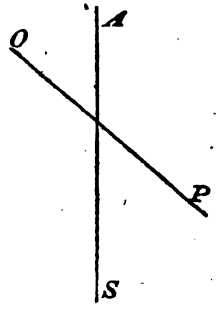
of force S, the force varying $\frac{1}{D^2}$; in its descent, it impinges upon a hard plane OP inclined to SA at a given angle, and after describing a certain curve comes to the plane on the other side, and is then reflected to the center; find the nature of this curve; and determine the whole time of descent to the center S in terms of the periodic time of a body revolving in a circle at the distance SA.



11. Let parallel rays be refracted through two contiguous double convex lenses; find the focal length on the supposition that the radii of all the surfaces are equal, and the sine of incidence : sine of refraction :: 5 : 4.

12. Given the latitude of the place and the declination of the sun, the former being less than the latter; to find at what time of the day the shadow of a stick would be stationary, and how far it would afterwards recede on the horizontal plane.

13. Transform the equation $x^n - px^{n-1} + qx^{n-2} -$ &c. $= 0$ into one, whose roots are the reciprocals of the sum of every $n-1$ roots of the original equation.

14. A body descends down the cycloidal arc AM, the base AL being parallel to the horizon and M the

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lowest point of the cycloid; determine that point where its velocity in a direction perpendicular to the horizon is a maximum.

15. Construct the equation $a^2y^2 - x^2y - a^3 = 0$.

16. Compare the time of descent to the center in the logarithmic spiral with the periodic time in a circle, whose radius is equal to the distance from which the body is projected downward.

17. Given the difference of altitudes of two stars, which are upon the meridian at the same time, and their difference of altitudes and difference of azimuths an hour afterwards, to find the latitude of the place.

18. A person's face in a reflecting concave decreases to the principal focus, and then increases in going from it.—Required a demonstration.

19. Prove that the mean quantity of the disturbing force of S upon P, in the 66th proposition of Newton, during one revolution of P round T, is ablatitious, and equal to half the mean addititious force.

20. The time of the sun's rising is the time which elapses between the appulse of the upper and under limb of the sun's disc to the horizon; given the sun's apparent diameter and the latitude of the place, it is required to determine the declination, when this time is a minimum.

21. Through a given point situate between two right lines given in position, to draw a third line cutting them in such a manner, that the rectangle under the parts intercepted between the point and the two lines may be a minimum.

22. Let a spherical body descend in a fluid from rest; having given the diameter of the sphere, and its specific gravity with reference to that of the fluid,

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it is required to assign the velocity of the sphere at any given point of the space described.

23. The distance of the center of gravity from the vertex of a solid formed by the revolution of a curved surface is $\frac{2}{3}$ of its axis.—Determine the nature of the generating curve.

24. Suppose a given cylindrical vessel filled with water to revolve with a given angular velocity round its axis.—Required the quantity contained in the cylinder, when the water and cylinder are relatively at rest.

25. Sum the following series:

$\frac{10}{1.2.3.4} + \frac{14}{2.3.4.5} + \frac{18}{3.4.5.6}$ + &c. to n terms and ad inf.

$\frac{5}{1.2.3} \times \frac{1}{2^2} + \frac{6}{2.3.4} \times \frac{1}{2^3} + \frac{7}{3.4.5} \times \frac{1}{2^4}$ + &c. ad inf.

$\frac{1}{1.3} \frac{1}{3.5} + \frac{1}{5.7}$ &c. ad inf.

26. Given the fl. $\overline{a+cz^n}\vert^m \times z^{pn+n-1}\dot{z}$ to find the fl. $\overline{a+cz^n}\vert^{m+1} \times z^{pn-1}\dot{z}$;

Required also fl. $\frac{\dot{x}\sqrt{a^2+x^2}}{x^3}$; and of $\frac{z^\theta \dot{z}}{1+mz}$, θ being a whole positive number.

Morning Problems.—Mr. Dealtry.

FIRST AND SECOND CLASSES.

1. Inscribe the greatest cone in a given spheroid.

2. A parabolic surface is immersed vertically in a fluid, whose density increases as the depth, with

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its base contiguous to the surface of the fluid; find upon which of the ordinates to the axis there is the greatest pressure.

3. Solve the equation $x^3 - px^2 + qx - r = 0$, whose roots are in geometric progression.

4. Suppose the reflecting curve to be a circular arc, and the focus of incident rays in the circumference of the circle, to find the nature of the caustic.

5. If the sine of incidence : sine of refraction :: m : n, required the focal length of a hemisphere, the rays falling first on the convex side.

6. If the subtangent of a logarithmic curve be equal to the subtangent of the reciprocal spiral, prove that the arc intercepted between any two rays in the spiral is equal to the arc intercepted between any two ordinates of the curve respectively equal to the former.

7. In what direction must a body be projected from the top of a given tower with a given velocity, so that it may fall upon the horizontal plane at the greatest distance possible from the bottom of the tower?

8. Draw an asymptote to the elliptic spiral.

9. If water or any fluid ascends and descends with a reciprocal motion in the legs of a cylindrical canal inclined at any angle, to find the length of a pendulum which will vibrate in the same time with the fluid.

10. Find the fl. $vx\dot{x}$, where $v = H.L.\overline{x + \sqrt{x^2 + a^2}}$

11. The centrif. force at the equator arising from the rotation of the earth round its axis : the centrifugal force in any parallel of latitude :: $\overline{\text{rad.}}^2 : \overline{\text{sine}}^2$ of the co. lat.—Required a proof.

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12. Given the latitudes of two places together with their difference of longitudes, to find the declination of the sun, when it sets to the two places at the same time.

13. Required the equation to a curve, whose subtangent is equal to n times its abscissa.

14. If the force vary $\frac{1}{D^{n+1}}$, how far must a body fall externally to acquire the velocity in any curve, whose chord of curvature at the point of projection is c; and apply the expression to the parabola and logarithmic spiral.

Afternoon Problems.—Mr. Palmer.

THIRD AND FOURTH CLASSES.

1. Find the value of £.123333 &c.

2. Determine geometrically a mean proportional between the sum and difference of two given straight lines.

3. What is the general form of parallelograms, whose diameters cut each other at right angles?

4. Investigate the area of a circle, whose diameter is unity: and prove that the areas of different circles are in a duplicate ratio of their diameters.

5. Divide a given line into two parts, such that their product multiplied by their difference may be a maximum.

6. Prove that in any curve the velocity : velocity in a circle at the same distance (SP) : : $\sqrt{\text{chord of curvature}} : \sqrt{2SP}$.

7. A body projected from one extremity of the diameter of a circle, at an angle of 45°, strikes a

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mark placed in the center. Required the velocity of projection and greatest altitude.

8. Find the area of a curve whose equation is $y=\frac{a^3}{a^2-x^2}$.

9. In how many years will the interest due upon £100. be equal to the principal, allowing compound interest?

10. Admitting the periods of the different planets to be in a sesquiplicate ratio of the principal axes of their orbits, shew that they are attracted towards the sun by forces reciprocally proportional to the squares of their several distances from it.

11. Prove that in the course of the year the sun is as long above the horizon of any place as he is below it.

12. Determine the limits within which an eclipse of the sun or moon may be expected: and shew what is the greatest number of both which can happen in one year.

13. Prove that the time in which any regular vessel will freely empty itself: time in which a body will freely fall down twice its height :: area of base: area of orifice.

14. Find the fluents of $\frac{x\dot{x}}{\sqrt{a-x}}$; $\frac{x^2\dot{x}}{a-x}$.

15. Find the principal focus of a lens; and shew how an object may be placed before a double convex lens, that its image may be inverted and magnified so as to be twice as great as the object.

16. Prove that Cardan's rule fails unless two roots of the proposed cubic be impossible; and determine

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whether that rule be applicable to the equation $x^3-237x-884=0$.

17. Deduce Newton's general expression in sect. 9. for the force in the moveable orbit.

18. Define logarithms, and explain their use: also, prove that log. $A \times B = \log A + \log. B$.

19. Explain the different kinds of parallax; and shew from the want of parallax in the fixed stars, that their distance from the earth bears no finite ratio to that of the sun.

Afternoon Problems.—Mr. Palmer.

FIFTH AND SIXTH CLASSES.

1. How many yards of cloth, worth 8s. 7½d. per yard, must be given in exchange for 935½ yards, worth 18s. 1½d. per yard?

2. Find the interest of £873. 15s. 0d. for 2½ years, at 4½ per cent.

3. Prove that the diameters of a square bisect each other at right angles.

4. Prove the opposite angles of a quadrilateral figure inscribed in a circle equal to two right angles.

5. Prove that if $A \propto B$ when C is given, and $A \propto C$ when B is given, when neither B nor C is given, $A \propto BC$.

6. Prove radius a mean proportional between tangent and cotangent; and that sine × cosine $\propto$ $\overline{\text{sine}}^2$ of twice the angle.

7. Given the sine of an angle, to find the sine of twice that angle.

8. Prove that in the parabola $\overline{\text{ordinate}}^2 =$ abscissa × parameter.

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9. Extract the square root of $a^3 - x^3$.

10. Solve the equation $3x^2 - 19x + 16 = 0$.

11. Prove that motion when estimated in a given direction is not increased by resolution.

12. Find the ratio of $P : W$ when every string in a system of pullies is fastened to the weight.

13. Prove that time of oscillation $\propto \frac{\sqrt{\text{length}}}{\sqrt{\text{force}}}$.

14. Prove that when a fluid passes through pipes kept constantly full, velocity $\propto \frac{1}{\text{area of section}}$.

15. Define the center of a lens; and find the center of a meniscus.

16. Find the fluxion of $\sqrt{a^3 + x^3} - \sqrt{a^2 - x^2}$.

17. Prove elevation of the æquator above the horizon = co. latitude.

18. Prove that sagita $\propto \overline{\text{arc}}^2$.

19. Prove that in the same orbit velocity $\propto \frac{1}{\text{perp}}$.

Evening Problems.—Mr. Palmer.

FIRST, SECOND, THIRD, AND FOURTH CLASSES.

1. When £100. stock may be purchased in the 3 per cents. for £59½. at what rate may the same quantity of stock be purchased in the 5 per cents. with equal advantage?

2. A ball of wood being balanced in air by the same weight of iron, how will the æquilibrium be affected when the bodies are weighed in vacuo? and by what weight of wood, properly disposed, may the equilibrium be restored?

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3. Investigate the value of the circumference of a circle whose radius is unity.

4. Compare the areas of the parabolas described by two bodies projected together from the same point, and with the same velocity, towards a mark situated in an horizontal plane, the angles of elevation being to each other : : 2 : 1.

5. Prove the rule for finding the quadratic divisors of any equation; and apply it to the equation $x^4 - 17x^3 + 88x^2 - 172x + 112 = 0$.

6. On what point of the compass does the sun rise to those who live under the æquinoctial, when he is in the northern tropic?

7. How many equal circles may be placed around another circle of the same diameter, touching each other and the interior circle?

8. Determine the resistance of the medium in which a body by an uniform gravity may describe a parabolic orbit?

9. Prove that a body moving in the reciprocal spiral, approaches or leaves the center uniformly.

10. Find the velocity and time of flight of a body projected from one extremity of the base of an equilateral triangle, and in the direction of the side adjacent to that extremity towards an object placed in the other extremity of the base.

11. Define similar curves; and prove that conterminous arcs of such curves have their chords of curvature at the point of contact in a given ratio.

12. Compare the time of a revolution about the center of a given ellipse, with that about its focus.

13. Find the attraction of a corpuscle placed in the axis of a cylindrical superficies, whose particles attract in an inverse duplicate ratio of the distance.

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14. Prove that if the center of oscillation of a pendulum be made the point of suspension, the former point of suspension becomes the center of oscillation.

15. Determine the content of the solid generated by a semicircle revolving about a tangent parallel to it's base.

16. Find the fluents of

$$\frac{\dot{x}\,\overline{x}^{2}}{\sqrt{a-x}};\quad \dot{y}\,\overline{y}^{2n-1}\sqrt{a^n-y^n};\quad \frac{\dot{v}v}{a+v\,.\,\sqrt{a^2+v^2}}.$$

17. Sum the series $1-\frac{1}{2^3}+\frac{1}{2^5}-\frac{1}{2^7}+$ &c. ad inf. and also to n terms.

$\frac{1}{1.5}+\frac{1}{2.6}+\frac{1}{3.7}+$ &c. to n terms. $\frac{1}{1.3}+\frac{1}{3.7}+\frac{1}{5.11}$ + &c. ad inf.

18. Required the sun's place in the ecliptic, when the increment of his declination is equal to that of his right ascension.

19. Prove that the force by which a body may describe a curve, whose ordinates are parallel, is proportioned to $\pm\ddot{y}$: and determine the quantity q such that force $=q\times\pm\ddot{y}$.

20. Compare the times in which a cylinder, whose axis is parallel to the horizon, will discharge the first and last half of its content through an orifice in its lowest section.

21. Prove that the image of a straight line immersed in water appears concave to an eye placed any where between the extremities of the line.

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22. At what distance from the earth would the apparent brightness of the moon be equal to that of Saturn and his ring together, supposing the apparent brightness of Saturn to that of his ring : : 2 : 1?

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Morning Problems.—Mr. Hornbuckle.

FIRST AND SECOND CLASSES.

1. Find six geometric means between the numbers a and b.

2. Half the breadth of any very thin glass lens is a mean proportional between its thickness and focal length nearly. The demonstration is required.

3. Find the integral of $\frac{x}{4}$, x being unity.

4. Sum the series $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+$&c. from the series expressing the cosine of a given arc in terms of the arc.

5. In what part of the moon's orbit is her gravity towards the earth unaffected by the action of the sun?

6. A globe of given weight is supported between two planes inclined to the horizon at the respective angles of 60° and 30°. Compare the weights sustained by the planes with each other and with the whole weight.

7. Shew that a particle in the vertex of a triangle, from which a perpendicular falls on the base, is attracted towards the segments of the base with forces which are to each other inversely as the secants of the angles at the base: the law of attraction being the inverse square of the distance.

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8. Two stars, whose right ascensions and declinations are given, are on the same azimuth, and the altitude of one of them is known; the latitude of the place is required.

9. Determine geometrically those points in an ellipse in which the centripetal and centrifugal forces are equal, the center of force being in the center of the ellipse.

10. Find the time of vibration of a cylindrical rod of given length suspended at one end, the density of which varies as the distance from the point of suspension.

11. Into how many parts must a given quantity, 2, be divided, that their continued product may be a maximum?

12. The altitude of the mercury in the gauge of a receiver is a inches, the standard altitude being h inches, after n turns: compare the capacities of the receiver and the barrel.

13. A ring of given weight descends by its gravity down the arc of a given quadrant which revolves uniformly about its axis perpendicular to the horizon in t''. Find the velocity of the ring at any point of its descent.

Afternoon Problems.—Mr. Woodhouse.

THIRD AND FOURTH CLASSES.

1. Of right-angled triangles having the same base, determine that, down whose hypothenuse, a body descends in least time.

2. Compare the angular velocities of SP and Sy, when a body moves in an ellipse, force in focus.

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3. What is the space through which a sphere sinks in water: the specific gravity of sphere being $\frac{1}{2}$ of specific gravity of water?

4. From what arguments is it inferred that the earth's form is not spherical?

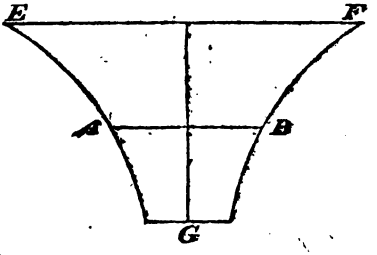
5. Required length of pendulum vibrating 180 times in a minute.

6. A and B hang over a pully, and $A=2B$: through what space will a body descend by the action of gravity, whilst A descends two feet?

7. To what height would a body moving in an ellipse round the center ascend, if projected upwards in the direction of a line from the center, with the velocity with which it revolves?

8. Explain the construction of the common pump.

9. What must be the form of the vessel EFG, so that the pressure on every horizontal section, as AB, shall be the same?



10. If parallel rays pass through a prism whose refracting angle is small, and the angle of incidence be also small, shew that the deviation is proportional to the refracting angle.

11. Find the expression for the force of the body in the moveable orbit, supposing the immoveable orbit to be an ellipse with force in center.

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12. The focal length of a sphere is n times the radius of the sphere. Required proportion of sines of incidence and refraction.

13. Draw a line to touch two given circles.

14. Find the area of an equilateral and equiangular octagon.

15. If a globe in its quantity of matter = a cylinder that could exactly circumscribe it, what is the proportion between the densities of the globe and cylinder?

16. Given the height *(h)* and radius *(r)* of a section of the outer surface of a cylindrical shell, to find the thickness of the shell, when its quantity of matter = quantity of matter in sphere (rad. r), (densities the same): shew also, than what line, h must necessarily be greater.

17. Give the definition of the fluxion of a quantity, and, from such definition, find the fluxion of $x^{\frac{m}{n}}$.

18. Find area of curve, the ordinate *(y)* of which $= ax + \frac{b}{\sqrt{1-x^2}}$.

19. Sum n terms of $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64}$, &c.

20. Shew how the logarithm of 101 may be found.

21. Which is greater, x^3+y^3, or x^2y+y^2x, $x > y$?

22. If from any point within an equilateral triangle, perpendiculars be drawn to the three sides, their sum equals a perpendicular drawn from one of the angles on the opposite side. Required proof.

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Evening Problems.—Mr. Woodhouse.

1. Two weights, $A=5$, $B=3$, hang over a pully: What must be the length of a pendulum that makes one oscillation, whilst A descends one foot?

2. In a cylinder filled with water to the height (h), place a sphere, whose specific gravity $=\frac{1}{2}$ of the specific gravity of water; and whose diameter very nearly equals that of the cylinder: What will be the ascent of the water?

3. Given the horizontal refraction (h), find how much the rising of the sun is accelerated by it.

4. Let three points with velocities, V, V', V'', move the same way, uniformly in the periphery of a circle. Required the time of their conjunction, supposing them to quit a given point at the same time.

5. Sun's meridian altitude on longest day $=63°$. 14′. Required latitude of place.

6. Determine the form of a surface, such, that rays issuing from a given point, may, after refraction, be made convergent to another given point.

7. Let a body be projected obliquely to the distance, with a force varying as n^{th} power of the distance: shew in what values of n the angle between the apsides will not be affected by the excentricity of the orbit.

8. A funipendulous body in an uniformly resisting medium is accelerated by a force varying as the distance from the lowest point in the curve. Required the point where the velocity is the greatest; and also, the time of one oscillation.

9. Given the apparent diameter of the sun at the mean distance, and the angular distance from apogee to find the sun's apparent diameter.

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10. A body with a velocity of 80 feet per second is projected upwards, along a plane 50 feet long, and inclined to the horizon at an angle of 30°: prove that the body, after quitting the plane, describes a parabola; and find the parameter.

11. The length of the solar year being $365^d.\ 5^h.\ 48'\ 49''$. shew that the intercalations may be determined from a series of fractions, as $\frac{4}{1}, \frac{29}{7}, \frac{33}{8}, \frac{128}{31}, \frac{161}{39}, \frac{2704}{655}, \frac{2865}{694}, \frac{5569}{1349}, \frac{86400}{20929}$; so that, the intercalation of 1 day in 4 years is too great, of 7 days in 29 years too little, of 8 days in 33 years too great, &c. &c.

12. If, on the inner surface of a paraboloid, a body whirled round as a circular pendulum (the point of suspension being in the axis of the paraboloid) describes a circle, then the time of revolution is the same, whatever is the circle, and equal to two oscillations of a common pendulum, the length of which equals the semiparameter of the parabola. Required proof.

13. What sum ought to be given for the lease of an estate for 20 years, of the clear annual rent of £100. in order that the purchaser may make 8 per cent. of his money.

14. $\text{Sin.}\, nA = 2\cos.A.\sin.(n-1)A - \sin.(n-2)A.$
$\cos.nA = 2\cos.A\cos.(n-1)A - \cos.(n-2)A.$
Required proof.

15. a, b, c, &c. being any quantities, make $\Delta a = b-a$, $\Delta^2 a = c-2b+a$, $\Delta^3 a = d-3c+3b-a$, &c. then, $a+bx+cx^2+\&c. = \frac{a}{1-x} + \frac{\Delta a.x}{(1-x)^2} + \frac{\Delta^2 a.x^2}{(1-x)^3} + \&c.$ Required proof.

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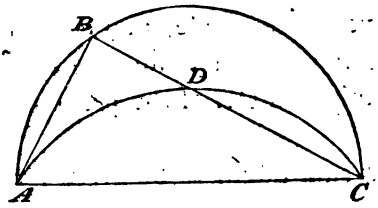
16. Find the integral equation of $ay+\frac{b\dot{y}}{\dot{x}}+\frac{c\ddot{y}}{\dot{x}^2}$ $=0$, a, b, c, being constant.

17. $(a+b)(a+b-1)(a+b-2)\ldots(a+b-n+1)$
$=a(a-1)(a-2)\ldots(a-n+1)+a(a-1)(a-2)$
$\ldots(a-n+2)\,nb$
$+a(a-1)(a-2)\ldots(a-n+3).\frac{n.n-1}{2}b.(b-1)$
$+$ &c.
$+b.(b-1)(b-2)\ldots(b-n+1)$. Required proof.

18. If a cylinder and inscribed sphere be cut by two parallel planes perpendicularly to the axis, prove that the surfaces of cylinder and of sphere included between the planes, are equal.

19. ABC is a semi-circle, ADC a quadrant: prove that the line BA always $=BD$; and that the longer only of the lines AB, CB, can cut the quadrant.



20. Let a, b, c, be the sides of any rectilinear triangle; put $S=a+b+c$; then the area is equal to the square root of this product, viz. $\frac{S}{2}\left(\frac{S}{2}-a\right)\left(\frac{S}{2}-b\right)\left(\frac{S}{2}-c\right)$. Required proof.

21. Let S', S'', S''', be the respective projections of any plane surface S, on three co-ordinate planes, then $S^2=S'^2+S''^2+S'''^2$. Required proof.

22. Shew that there can only be five regular polyhedrons.

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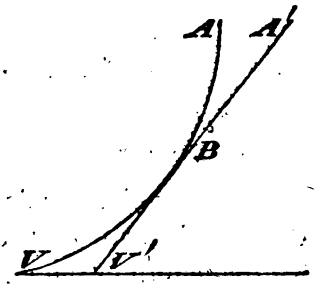
23. Of all isoperimetrical polygons having the same number of sides, the greatest is that which is equilateral. Required proof.

Morning Problems.—Mr. Woodhouse.

FIRST AND SECOND CLASSES.

1. The straight line $A'BV'$, = the semi-cycloid ABV, and touches it at the middle point B: compare the time of describing $A'BV'$ with the time of half an oscillation.



2. Let the reflecting curve be a circular arc, and let parallel rays be incident in its plane: find the cusp of the caustic, and prove the density of reflected rays to be there greater, than at any other point.

3. Two bodies from different distances fall to the center of force $\left(\text{force} \propto \frac{1}{(\text{dist.})^2}\right)$; then shew, that the times of their descent are in the sesquiplicate ratio of the whole spaces described.

4. Compare the times of emptying (on the common hypothesis) through equal orifices, two prisms of different heights and bases.

5. Prove that at the point where the $\angle SPy$ (in any curve) is a minimum, the angular velocities of

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SP and Sy are equal; and determine that point in an ellipse, force in focus.

6. If a body fall down the axis of a right-angled cone, the vertex downwards and axis vertical, in what point of its descent will it have acquired velocity sufficient to describe a circle on the surface of the cone, when whirled round as a circular pendulum?

7. The force varying as the distance, project a body from a given distance *(d)*, at a given angle *(a)*: investigate the orbit after the manner of the 8th section.

8. Tranform $x-\frac{x^2}{2}+\frac{x^3}{3}-$ &c. into a series as $\frac{ax}{1+x}+\frac{a'x^2}{(1+x)^2}+\frac{a''x^3}{(1+x)^3}+$ &c.

9. Find a number, of which divided by 2, 3, 5, respectively, the remainders are 1, 2, 3.

10. Determine the length of the curve, in which the rectangle of the abscissa and ordinate = the rectangle of the subtangent and a given line *(a)*.

11. Shew the method by which Archimedes approximated to the length of the circle.

12. Of all triangles having the same base and perimeter, the isosceles triangle is that of which the area is the greatest. Required proof.

13. The surface and solidity of a sphere are respectively equal to two thirds of the surface and solidity of the circumscribing cylinder, the areas of the bases of the cylinder forming part of its surface. Required demonstration.

14. Prove that the surface of a sphere can be completely covered with the surfaces either of 4, or of 8, or of 20 equilateral spherical triangles.

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Afternoon Problems.—Mr. Hornbuckle.

THIRD AND FOURTH CLASSES.

1. In what time will the interest of £1. amount to 15s. allowing $4\frac{1}{2}$ per cent. simple interest?

2. Solve the following equations $\frac{16}{x}-\frac{100-9x}{4x^2}=3$

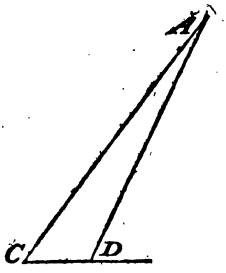
$$\left.\begin{aligned} x^2y+y^2x&=30\\ \frac{1}{x}+\frac{1}{y}&=\frac{5}{6}\end{aligned}\right\}$$

3. Extract the square root of $7-4\sqrt{3}$.

4. Find, by means of a given circle, two angles whose sines are in the proportion of 9 : 4.

5. Two bodies, A and a, are projected perpendicularly upwards with the velocities V and v: How high will their common center of gravity ascend?

6. Determine that part of the inclined plane AC through which a body will descend from rest in the time of describing AD.



7. Find the focal length of a meniscus whose thickness is inconsiderable.

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8. The reflecting curve is the logarithmic spiral and the radiating point in the center. Construct the caustic.

9. At what angle must the rudder of a vessel be inclined to the stream that the effect produced may be a maximum?

10. Prove $\dot{z} = F\dot{x}$, z being the space fallen through by the uniform force, 1, to acquire the velocity.

11. Prove the differential method by the method of increments.

12. A cylindrical tube, 40 inches long, is half filled with mercury and then inverted: How high will the mercury stand, the standard altitude being supposed 30 inches.

13. Required the velocity with which the air will rush into an exhausted receiver.

14. Find the fluents of $x\dot{x}\sqrt{a^2 - x^2}$, $\frac{\dot{x}}{x\sqrt{x-a}}$, $a^x\dot{x}$.

15. Investigate Cardan's rule for the solution of a cubic equation, and shew that it fails if all the roots are possible.

16. Shew that the general equation to the apsides can have but four possible roots.

17. Shew where an inferior planet is a morning or evening star.

18. Prove that when Aries rises the ecliptic makes the least angle with the horizon, and that this angle continually increases to its setting.

19. Render Newton's expressions for the law of force in his second section general.

20. If a body fall from a finite distance, and a given ellipse be described whose axis major is this distance, prove that the area described by the radius

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vector SD is equal to the area described in the same time in a circle at the distance of half the latus rectum of the ellipse from the same center.

Evening Problems.—Mr. Hornbuckle.

1. Find the equated time of payment of two sums S and s, due at T and t years hence respectively, allowing simple interest.

2. A body is projected from the top of a tower of a given height, at a given angle of elevation, and with a given velocity: find the range described on a plane drawn from the base of the tower at any given angle of inclination to the horizon.

3. The circumference of a given circle revolves uniformly about its diameter with a given angular velocity: compare the momentum of the circumference with that of those sides of the circumscribed square which are parallel to the axis of motion.

4. Suppose the axis major of a spheroid: the distance between the foci :: sine of incidence : sine of refraction, and that rays parallel to the major axis are incident on the convex surface; at that point in which any ray is incident draw a tangent to the ellipse, and a perpendicular to the tangent cutting the axis major; also join the point of incidence and each of the foci: it is required to prove, without any other construction, that the ray will be accurately refracted to the farther focus.

5. Sum the following series $\frac{1}{1.2}\times\frac{2}{3}-\frac{1}{2.3}\times\frac{3}{4}+\frac{1}{3.4}\times\frac{4}{5}-\frac{1}{4.5}\times\frac{5}{6}+$ &c. ad inf.

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and $\frac{4}{1.2.5}+\frac{5}{2.3.6}+\frac{6}{3.4.7}+$ &c. to n terms and to inf.

6. Given the apparent perpendicular depth of an object under the water, to find the direction in which a ball must be fired from a given situation to hit it, the path of the ball being supposed rectilinear.

7. Investigate the fluxional expression for the content of a solid, and apply it to find the content of that, which is generated by the revolution of a curve about its axis, whose equation is $y=$hyp. log. $(1+x,)$ x being the abscissa, and y the corresponding ordinate.

8. Prove that the sum of the infinite series $\frac{1}{1^3}-\frac{1}{3^3}+\frac{1}{5^3}-$ &c. $=\frac{2^3}{4}$, 2 being a quadrantal arc to radius 1.

9. The sum of the m^{th} powers of the roots of the equation $x^n-1=0$ is n, if $\frac{m}{n}$ be any whole number; and 0, if it be a fraction. Required a proof.

10. Compare the space described in one second by the force of gravity in any given latitude, with that which would be described in the same time, if the earth did not revolve round its axis.

11. At the extremities of the arms of a given lever are suspended respectively the weights p and q; supposing p to descend, and the lever to revolve round the fulcrum; it is required to determine the angular velocity of the lever at any point of its descent from an horizontal to a vertical position; the inertia of the lever being neglected.

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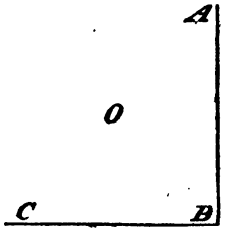
12. Required the length of the longest ladder that can be slided up a wall AB from the horizontal plane CB, under an obstacle O given in position.



13. Prove the diurnal path of the shadow of any object perpendicular to the horizon in all latitudes a straight line, when the sun is in the equinox.

14. The refracting curve is the logarithmic spiral, and the radiating point in the center; construct the caustic.

15. A flexible string of given length is wrapped round a cylinder whose weight and dimensions are given; one end of it being fixed to the surface of the cylinder, and the other to a tack; if the cylinder be suffered to descend by its gravity, it is required to find the time in which the whole string is unwound, and the velocity acquired.

16. Given the force of gravity on the earth's surface; the periodic time of the moon, and her mean distance: it is required to compare the quantities of matter in the moon and earth.

17. Find the fluents of $\frac{a^2 x^{-\frac{1}{2}} \dot{x}}{\overline{a+x}|^{\frac{1}{2}}}$, $\frac{x^n \dot{x}}{1+ax+x^2}$, n being a whole number.

18. Construct a vertical east dial.

19. A vessel of given dimensions in the shape of a paraboloid is filled with water, and revolves

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uniformly round an axis perpendicular to the horizon. Required the time of rotation that it may just be emptied.

20. Find the fluxional expression for the force tending in lines parallel to the axis of a curve when the oscillations therein are isochronous, and apply it to the catenary.

21. A flag-staff of given height is erected on a tower whose height is also given: at what point on the horizon will the flag-staff appear under the greatest possible angle.

22. Find the pressure sustained by a given horizontal plane; the height of the atmosphere being infinite, and its density being supposed uniform and n times as great as that at the earth's surface.

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1804.

Morning Problems.—Mr. Hudson.

FIRST AND SECOND CLASSES.

1. Point out the difference in principles, in exhaustions, indivisibles, prime and ultimate ratios, and the method of fluxions.

2. Find the integral of

$$\overline{x-m}.\overline{x-5m}+\frac{\overline{x-2m}.\overline{x-5m}}{2},$$ the increment of (x) being expressed by (m).

3. A and B play, by turns, 2 bowls with equal skill; A wants 3 of the game, and B 2; find their respective probabilities of winning.

4. Two elastic balls, beginning their motion from different points in the same right line perpendicular to an horizontal plane, are inflected along the plane with the velocities acquired, by a hard plane inclined to their direction at an angle of 45°; given the distance at which one impinges on the other on the horizontal plane, and the point from which one of them descended, to find the point from which the other began its motion.

5. With a given straight line perpendicular to the horizon, as radius, and the lower end as center, describe a circle, and draw a tangent at its extremity; if any radius of this circle represent the position of

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an inclined plane, and a body be projected on it with the velocity acquired in falling through the given line, the space described on this plane in the time of its descent will be equal to the radius of the circle, together with the perpendicular drawn from the intersection of the plane and the circle to the tangent.

6. Let the atmosphere be supposed to consist of equally elastic particles, which at different distances repel each other with forces inversely proportional to the squares of the distances between their centers, and let the tube of a barometer be partly filled with mercury, and then its open end immersed in a bason of the same fluid; investigate an equation for finding the height of the mercury in the tube.

7. At a certain place in a given N. latitude, the sum of the sun's declination and altitude was observed at a given hour in the morning; find the sun's altitude and declination.

8. Let a small reflector be moved along an horizontal plane with its surface parallel to it, and let an object move in a straight line perpendicular to the plane, and so as to be always at the same distance as at first from the reflector; find the locus of an eye, which being always equidistant from the reflector, shall see the image.

9. Three known rectilinear objects placed contiguous, and in the same right line, appeared to the eye of a spectator of the same length; find his position.

10. Let a body be projected at an angle of 45° with the horizon, and suppose the arc of the earth's surface intercepted between the points of projection

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and incidence to be 60°; find the velocity of projection.

11. Let the earth be supposed a sphere of given magnitude, and to revolve about its axis in a given time; compare the weight of a body at its equator with its weight in a given latitude.

12. Investigate the rule for finding by fluxions the center of oscillation of any line, surface, or solid

13. Find the solid content of a section of a given spheroid made by a plane coincident with the ordinate of a given diameter, and cutting the diameter at a given distance from the center.

14. Find the locus of the vertex of a triangle described on a given base; 1. when the sum of the angles at the base is given: 2. when one of them is always double the other.

Morning Problems.—Mr. Woodhouse.

FIRST AND SECOND CLASSES.

1. Required two geometric means between 10, and 100.

2. $b=\frac{a-\sqrt{a^2-x^2}}{a+\sqrt{a^2-x^2}}$. Required x.

3. Prove that tang. $\frac{A}{2}=\frac{\text{sin. } A}{1+\text{cos. } A}$ (radius 1).

4. Prove that if the difference between the digits of the odd and even places in any number be adjoined to the number, the result is divisible by 11.

5. Suppose a square and its circumscribing circle to revolve round a diagonal of the square; compare the surfaces and solidities of the solids generated.

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6. Shew how the circumference of an ellipse may be computed, of which the excentricity is .99, the semi-axis major being 1.

7. Given the force at earth's surface, earth's radius, and moon's distance, to find the moon's periodic time.

8. Find the fluent of

$$\frac{x^2\dot{x}}{(1+x^2)^{\frac{1}{2}}}$$

and prove $\int \frac{x^5\dot{x}}{\sqrt{1-x^2}}$, contained between the values of x, 0 and 1, to equal $\frac{2.4}{3.5}$.

9. Explain the principle on which achromatic telescopes are constructed.

10. If parallel rays fall on equal aperture of spherical reflectors, the lateral aberrations vary inversely as the squares of the radii of the reflectors. Required proof.

11. What sum of money ought to be paid every three years instead of £100. paid annually, the rate of interest being 5 per cent.?

12. Shew that the rectification of the hyperbola may be deduced from that of the ellipse.

13. Shew that if the moon's velocity were increased, her synodic revolution would differ less from her sidereal.

14. In the lemniscata, whose equation is $(x^2+y^2)^2 = x^2-y^2$, it is required to assign two arcs equal to one another.

Afternoon Problems.—Mr. Hudson.

THIRD AND FOURTH CLASSES.

1. In a system of notation whose local value is (a), in any multiple of $(a-1)$ the sum of the digits is either equal to $(a-1)$, or a multiple of it.

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2. Any power of a cube root of unity, is itself a cube root of unity.

3. If two circles touch each other, any straight line passing through the point of contact cuts off similar parts of their circumferences.

4. If from the angular points of any plane triangle, three straight lines be drawn through the same point within it to the sides, 6 segments will be formed; shew that the solids, measured by 3 that are not contiguous, are equal.

5. Given the tangents of two arcs of a given circle, to find the tangents of the arcs which are equal to their sum, and difference.

6. A person wishing to determine the height of an obelisk standing on a declivity, measured a given distance from its base, and took its elevation; and going on in the same direction a given distance, took its elevation again; shew how the altitude may be found from these data.

7. Given the perimeter, the vertical angle, and the perpendicular in a plane triangle, to construct it.

8. Solve the equations,

1. $3x^{\frac{5}{3}}+x^{\frac{5}{6}}=3104.$
2. $x^3-\overline{a+p}.x^2+\overline{.ap+q}.x-aq=0.$
3. $x^2+xy=10.$
   $y^2+yz=21.$
   $z^2+xz=24.$

9. The evolute of a common cycloid is another equal cycloid.

10. On a given straight line perpendicular to the horizon, describe a circle, and draw a tangent at its higher extremity; if any point be assumed in this tangent, and a straight line be drawn from it parallel

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the diameter, cutting the circle in two points, and chords be drawn from those points perpendicular to the diameter, the times of describing the parts of this line, together with the times of describing the corresponding chords with the velocities acquired at their extremities, shall be equal.

11. Let the compressive force be supposed proportional to the square of the density, and the force of gravity inversely proportional to the square of the distance from the earth's center; it is required to find the law of the density in the atmosphere.

12. Given the azimuth and altitude of the sun at two different times during the same morning, to determine the latitude of the place.

13. Let the radii of the surfaces of a double convex lens of known refracting power be given, and the positions of the eye and object in the axis of the lens; to compare the apparent magnitudes of the object and image.

14. To an eye that moves in the circumference of a given circle, find where a known rectilinear object given in position in the same plane with the circle will appear a maximum, and where a minimum.

15. Let a weight appended at the circumference of a wheel elevate by its gravity a weight appended at the axle; given the two weights, the weight of the wheel, and the radius of the axle, to find at what distance one of the weights must be applied, so as to raise the other at the axle through a given space in the least time.

16. A body having descended from a given altitude, begins to descend along the arc of a given cycloid whose base coincides with an horizontal plane;

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find the points where it will leave the curve, and where it will meet the horizontal plane.

17. Find the ratio of the times of oscillation of a pendulum, at the equator, and at the pole, supposing the earth to be a sphere, and to revolve about its axis in a given time.

18. If the first fluxions of a common ordinate of a curve and a circle be equal, they will have the same tangent at that point; and if the second fluxions be equal, it will be impossible to describe another circle that shall approach nearer to the curve.

19. If (x) increase in geometrical progression, the successive increments of the ratio of (x) to (1) will be equal.

20. Of all cones, of the same solid content and specific gravity, to determine the ratio of the diameter of the base to the altitude in that, which, when immersed with its axis perpendicular to the surface of a given fluid, shall have the surface immersed, a minimum.

21. A body revolving in a given circle, acted on by a force, which is inversely proportional to the square of the distance from the center, in consequence of an impulse in the direction of its motion, begins to describe an orbit of given excentricity; find the velocity communicated to it by the impulse.

22. The given circle described by a body, acted on by a force tending to a point in the periphery, is caused to revolve about the center of force in such a manner that the angular velocity of the body describing it, is to that of a body describing the same orbit when quiescent, in a given ratio; find the equation to the orbit described in fixed space, the law of

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force, and velocity of projection at the apsis, by which a body might describe it.

23. Find the point in the moon's orbit, which is here supposed circular, when that part of the ablatitious force which accelerates her motion is a maximum.

24. If two bodies attract each other with forces inversely proportional to the square of the distance between their centers, and be projected in parallel and opposite directions with given velocities, they will describe about their common center of gravity similar figures.

Afternoon Problems.—Mr. Woodhouse.

THIRD AND FOURTH CLASSES.

1. The interior angles of every quadrilateral figure are equal to four right angles. Required proof.

2. Reduce .855 of a foot, to the decimal of a yard.

3. Given the logarithms of 2 and 3, to find the logarithm of 216.

4. What 3 numbers are those that have their differences equal, their sum 15, and sum of cubes 495?

5. Sum $\frac{1}{1}+\frac{1}{4}+\frac{1}{10}+\frac{1}{20}+$&c.

6. Prove that sin. $(a+b)$. sin. $(a-b) = (\text{sin. } a)^2 - (\text{sin. } b)^2$.

7. Shew how the focus of a concave lens may practically be found.

8. Explain the principle and use of the hydrostatic balance, and determine the specific gravity of a

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piece of silver that weighs 135 grains in air, and 123.3125 in water.

9. Required center of gravity of three equal bodies placed at the three angles of an isosceles right-angled triangle.

10. Shew that there is no finite distance through which a body at rest can fall, so as to acquire the velocity in the logarithmic spiral, the force $\propto \frac{1}{D^5}$.

11. Let a ray of light fall obliquely on a plane refracting surface: determine the position of the refracted ray, the ratio of the sines of incidence and refraction being given.

12. From what argument is it probable that the moon has no atmosphere?

13. Shew the variation of the earth's angular velocity to be nearly twice as great as it would have been, had the earth's motion been uniform.

14. Compare the apparent diameters of the sun at mean and least distance.

15. Two bodies projected along two planes inclined to the horizon at angles 45°, and 30°, describe spaces respectively as $\sqrt{2} : \sqrt{3}$. Required the ratio of the initial velocities of the projected bodies.

16. Let two equal bodies from the vertex of an isosceles triangle, the vertical angle being 120°, move uniformly along the sides; compare the space described by the center of gravity with the space described by either body.

17. Compare the apparent diameters of the sun seen from the earth and Jupiter.

18. In 39th proposition find the area $ATVME$ when $DF = a$, a being a given line.

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19. What would be the proportion between the axes of the earth's orbit, if the apparent diameter of the sun at mean distance were to his apparent diameter at perigee :: 2 : 5?

Afternoon Problems.—Mr. Hudson.

FIFTH AND SIXTH CLASSES.

1. If the number (2) be divided into any two parts, the difference of the parts is equal to the difference of the numbers, formed by adding each to the square of the other.

2. Solve the equations,

1. $\frac{2x-3}{3}-\frac{10x-4}{18}=-1.$

2. $x^{\frac{6}{5}}+x^{\frac{3}{5}}=756.$

3. $\overline{x+2}^2+\overline{x+2}=20.$

4. $x+y+\sqrt{xy}=19.$
$x^2+y^2+xy=133.$

3. The space described by a heavy body in the 4th second, is to the space described in the last second except 4, as 1 to 3; required the whole space described.

4. Let a cubical vessel be filled, half with water, and half with mercury; find the ratio of the pressure on the sides to the pressure on the base.

5. Of all triangles described on the same base, and having the same altitude, the perimeter of the isosceles is least.

6. If the abscissa of an algebraic curve increase uniformly, the deflection from the tangent is ultimately equal to half the quantity that measures the second fluxion of the ordinate.

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7. A body is projected at a given point with a given velocity, acted on by a force which varies inversely as the square of the distance from the center; compare the chord of curvature at the point of projection with the distance.

8. A body moves through a given chord of a great circle of the earth in a tube, acted on by a force which is at every point proportional to its distance from the earth's center; find the ratio of the whole time of describing the tube, to the periodic time of a body revolving in a circle at the earth's surface.

9. Given the base, the vertical angle, and the perpendicular, in a plane triangle, to construct it.

Afternoon Problems.—Mr. Woodhouse.

FIFTH AND SIXTH CLASSES.

1. At $3\frac{1}{2}$ per cent. to find the interest of £3. 15s. 4d. for one year.

2. It is required to inscribe in a circle an equiangular and equilateral octagon.

3. Find the time of describing 30 feet on a plane inclined to the horizon at an angle of 30°, the force of gravity being supposed to be diminished by one-fourth of its present quantity.

4. Shew that the velocity of the earth continually increases from the mean to the perihelion distance.

5. Which is greater, $\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+$ &c. ad inf.

or $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+$ &c. ad inf.?

6. Prove that $\cos. 2a = 2(\cos. a)^2 - 1$ (radius $= 1$).

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7. Find the focal length of a plano-convex lens, the sine of incidence being to the sine of refraction :: 1,5 : 1.

8. Find the fluxion of $\frac{x}{\sqrt{1+x^2}}$, and the fluent of $\frac{\dot{y}}{(1+y^2)^{\frac{1}{2}}}$.

9. Compare the pressure on the inner surface of a cylinder filled with water, with the weight of the fluid.

10. The emersion of a satellite of Jupiter is observed to happen at a certain place at 11*h*. 30′. apparent time, and at Greenwich it is calculated to happen at 10*h*.; find the longitude of the place with respect to Greenwich.

Evening Problems.—Mr. Hudson.

FOR THE FIRST SIX CLASSES.

1. Solve the equations,

$$3x^{\frac{4}{3}} - \frac{5x^{\frac{2}{3}}}{2} = -592.$$

$$\overline{x-4}|^2 + 2.\overline{x-4} = \frac{2}{x} - 1$$

2. Having given the sum and product of two algebraic quantities, investigate the series expressing the sum of their n^{th} powers.

3. Find the sum of the infinite series,

$$\frac{1}{1.2} - \frac{3}{4.5} + \frac{5}{7.8} - \frac{7}{10.11} \text{ \&c.}$$

and n terms of the series,

$$2+6+14+30+62+126 \text{ \&c.}$$

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4. Find the following fluents,

$$\frac{a^2\dot{x}}{\overline{a+x}.\overline{a-x}\,|^{\frac{3}{2}}}$$; and

$$\frac{z\dot{x}}{\sqrt{2rx-x^2}}$$; where z = circular arc whose rad. $= r$, and ver. sine $= x$.

5. Solve the fluxional equation $\dot{x}+x^3\dot{x}=\dot{z}+z\dot{x}$.

6. A person holding two common dice, undertakes to throw both the numbers 5 and 7, at 3 trials; find the odds against him.

7. Find the center and diameter of a circle that touches 3 given circles, each of which touches the other two.

8. If a spheroid be cut by a plane, the section will either be an ellipse or a circle, and all parallel sections will be similar figures.

9. A person standing in the center of a given circle, throws an elastic ball with a given velocity along a plane coinciding with the horizontal radius which is reflected to him by a small hard plane coinciding with the tangent; if the plane be conceived to revolve about the center of the circle, find in fixed space the locus of his position on the plane, when the ball projected with the same velocity, and reflected by the tangent-plane, always returns to his hand in the same time.

10. A ball half elastic having fallen from a given point, at the middle of its descent is projected parallel to the horizon with the velocity acquired; find the position of a hard plane, which, opposed perpendicularly to its motion, will cause it to move to the same point to which it would have descended in the right line, and find the whole time of its motion.

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11. To determine the length of a pendulum which, beginning an entire oscillation in a common cycloid from a given point, shall arrive at a given perpendicular to the horizon in less time than any other, beginning an oscillation from the same point.

12. A given parabolic conoid is put into a fluid, and when it is in a position of permanent equilibrium the base is wholly extant, and inclined to the horizon; having given its specific gravity relatively to that of the fluid, it is required to determine the dimensions of the part immersed.

13. The solid described in the last question being excavated, and containing a fluid, has its axis inclined to an horizontal plane till the surface of the fluid passes through one extremity of its base; in this position a hole being made at the extremity of that diameter which passes through the center of the fluid's surface, and the greatest perpendicular distance of the surface from the base being given, it is required to determine the time in which it will be emptied.

14. At any place situated between the equator and the poles, it is required to determine the two hours in the morning, during which the increase of the sun's altitude is greatest and least, on that day when he is in the equinoctial point.

15. At a given place in N. latitude, two stars, the difference of whose right-ascensions is known, are observed to be on the prime vertical at the same time; in this position their distance being observed, it is required to determine their declinations.

16. Given the sun's altitude and declination, and the sum of the azimuth and hour-angle, to determine the latitude of the place.

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17. Let a ring of given diameter be held with its plane perpendicular to the surface of a large plane reflector, and let a straight line be drawn from its center perpendicular to its plane; having given the position of the eye in this line, it is required to determine the position and apparent magnitude of the image.

18. Let the surface of a plane reflector be always perpendicular to a line which revolves about one of its extremities, and cuts two other lines given in position; it is required to determine the locus of the reflector, so that an object moving in the intersection of the revolving line with one of the given lines, the image shall move in its intersection with the other.

19. Let the position of the axis of a spherical surface of known refracting power, perpendicular to, and bisecting, a very distant object, be given, and in it the position of the eye and image, and also the apparent magnitudes of the object and image; to determine the magnitude and position of the refracting surface.

20. A body is projected in a given direction, at a known distance from an horizontal plane, with a given velocity, acted on by a force perpendicular to the plane, which is inversely proportional to the square of its distance from the plane; construct the orbit it will describe.

21. It is required to compare the polar and equatorial diameters:—1st. When the length of a degree of latitude, in each of two known places, is given. 2. When the length of a pendulum vibrating seconds, in each of the same two places, is given.

22. Two bodies whose weights are known, lying

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on a smooth horizontal plane, are connected by a flexible line passing through a small ring fixed at a given point between them; in this position a given velocity is communicated to one of them in a direction perpendicular to the line that joins their centers, and the other is made to move directly towards the ring: investigate the motion of the projected body, and find the angle described when the other body arrives at the ring.

Evening Problems.—Mr. Woodhouse.

1. What is the interest of £315. 10s. for one year, at 4¼ per cent.?

2. Of all triangles that can be formed by two given sides containing a variable angle, the greatest is that in which the contained angle is a right-angle.

3. Find the present value of £20. to be paid at the end of every five years, at 5 per cent. rate of interest.

4. Prove that

$$\frac{\sin.(a-b)}{\sin.a.\sin.b}+\frac{\sin.(b-c)}{\sin.b.\sin.c}+\frac{\sin.(c-a)}{\sin.c.\sin.a}=0.$$

5. If any triangular number be multiplied by 8 and increased by 1, the result is a square number. Required proof.

6. In a triangular pyramid, the sum of the squares of the faces that are perpendicular to each other, equals the square of the remaining face.

7. With two dice, compare the chance of throwing at one throw, an ace, with the chance of throwing five points.

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8. If a body suspended by a string oscillates through a quadrant (the extremity of the quadrant being the lowest point,) the force stretching the string at the lowest point is three times that which is due to the weight of the body. Required proof.

9. In a circle, compare the time of falling down a chord of 60° to the extremity of a diameter perpendicular the horizon, with the time of the oscillation of a pendulum equal in length to the chord.

10. Under an exhausted receiver a sphere sinks to a depth $=\frac{3}{4}$ diameter. Required the alteration in the depth of the immersion, when the air (its specific gravity being 0.00122) is admitted.

11. Let parallel rays fall on a spherical refractor, determine by construction the position of the refracted ray, and shew that it cuts the axis in a point nearer the surface, than any other ray does less remote.

12. Let the distance between two thin and equal plano-convex glass lenses, be equal radius of convex surface, determine the focus of parallel rays after refraction at each lens.

13. The excentricity of the earth's orbit being small, the variation of the angular velocity is nearly proportional to the cosine of the angle made by the radius vector and perihelion distance. Required a proof.

14. From the poles to the equator, the decrease of the length of a pendulum always vibrating in the same time, varies as the square of the cosine of latitude. Required proof.

15. If the altitude of a wave compared with its breadth be small, the wave moves over a space equal

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to its breadth, nearly in the same time that a pendulum, equal in length to the breadth of the wave, performs one oscillation. Required proof.

16. In the astronomical telescope, shew that, with a given object-glass, the magnifying power is increased, by increasing the convexity of the eye-glass; and explain what circumstances confine the increase of the convexity within certain limits.

17. In an elliptical orbit, if a be the semi-axis major, e the excentricity, u the angle contained between the radius vector and perihelion distance, then $r = \frac{a.(1-e^2)}{1+e.\cos.u}$. Required proof.

18. Find fluents of $\frac{\dot{x}}{x^3\sqrt{1-x^2}}$ and of $(\sin.z)^4\dot{z}$.

19. Prove that $\int \frac{\dot{x}}{\sqrt{1-x^4}}$ contained between the values of x, 0 and 1, $= \left(1-\frac{1}{2^2}+\frac{1^2.3^2}{2^2.4^2}-\frac{1^2.3^2.5^2}{2^2.4^2.6^2}+\right.$ &c. $\left.\right)\pi$, π being $=\frac{3.14159}{2}$.

20. Sin. $3a+\sin.5a+\sin.7a+$ &c. $\sin.(2m-1)a = \frac{\sin.(m+1)a.\sin.(m-1)a}{\sin.a}$. Required proof.

21. It is required to divide the quadrantal arc of an ellipse in such a manner, that the difference of the arcs may equal the difference of the semi-axes.

22. The sum of the squares of the coefficients of a binomial whose exponent is n, (n an integer,) equals the coefficient of the middle term of a binomial whose exponent is $2n$. Required proof.

23. Let two weights fastened to equal strings (l, l) move in a vertical plane, the one oscillating, the other

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revolving, and at the lowest points let the velocities due to heights h, h', be such, that $\frac{h}{2l}=\frac{2l}{h'}=m^2$, then the time of the revolution of one weight is to the time of oscillation of the other as m to 1. Required proof.

24. The periodic times of planets revolving in elliptical orbits are independent of the excentricities of those orbits. Required proof.

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1805.

First Morning.—Mr. Hornbuckle.

MONDAY, JANUARY 14, 1805.

1. The discount on a promissory note of £100. payable a year and a half hence, amounted to £7. 10s. What interest per cent. did the banker make of his money?

2. Prove that a table of sines and cosines constructed for 45°, will be sufficient to exhibit the sine or cosine of any arc in the circle.

3. Divide a given arc *(A)* less than a quadrant into two such parts, that the secant of one multiplied into the square of the secant of the other may be a maximum.

4. Find the principal focus of a plano-convex glass lens, and prove it a fixed point.

5. Suppose A=the sum of the roots of an equation, B the sum of the squares, C the sum of the cubes, &c.; required an expression equal to $A+B+C+$&c.

6. In a given latitude, it is required to determine the declination of the sun, that it may set in the S. W. point.

7. Two bodies, whose weights are 4lb. and 3lb. are projected together at an angle of 45° to the horizon, with the respective velocities of 19 and 12 feet

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in a second; required the path described by their common centre of gravity, and its greatest height above the horizontal plane.

8. Find the present worth of an annuity of *(A)* pounds, to begin *(n)* years hence, and continue *(t)* years at compound interest.

9. The refracting curve is the logarithmic spiral, and the radiating point in the center; construct the caustic.

10. Given the velocity of the wind, the angular motion of the sails of a windmill, and their inclination; required the length of the arms, that the effect produced by the wind may be the greatest.

11. Sum the following series $\frac{1.2}{2.3}-\frac{2.3}{3.4}+\frac{3.4}{4.5}-\frac{4.5}{5.6}+$ &c. and $\frac{10}{1.2.3.4}+\frac{14}{2.3.4.5}+\frac{18}{3.4.5.6}$ ad inf.

12. A given body impelled by the uniform force of gravity descends in a medium wherein the resistance varies as the velocity. It is required to exhibit the relation between the spaces, velocities, and times, by means of the logarithmic curve.

13. Find where the effect of the ablatitious force to accelerate a body's motion in its orbit is a maximum.

14. The distance between the 10 and 11 o'clock hour-line on a vertical east dial is 3 inches in length; required the height of the style.

First afternoon.—Mr. Hornbuckle.

FOR THE THIRD, FOURTH, FIFTH, AND SIXTH CLASSES.

1. If 7 horses eat 16 acres of grass in 20 days, how many will be required to eat 24 acres in 7 days?

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2. Add together $\frac{4}{7}$ of 5s. 6d., $\frac{2}{3}$ of $\frac{1}{5}$ of 17s. $2\frac{1}{4}$d., and $\frac{5}{7}$ of £5. 6s. $7\frac{1}{4}$d.

3. If a solid angle be contained by three plane angles, any two of these are greater than the third.

4. Prove that the sum of the tangents of two arcs is to the difference as the sine of the sum of these arcs is to the sine of the difference.

5. Solve the equation $x^3 - 3x^2 - 4x + 12 = 0$.

6. Find the quantity 2, so that $2.\overline{A^2 - B^2}$ may be a perfect c^{th} power.

7. Find the centre of gyration in a circle which revolves round an axis passing through its center perpendicular to its plane.

8. Find the radius of curvature at any point of the cycloid.

9. Find the time of flight of a body projected at an angle of 30° with a velocity of 193 feet in a second, the mark being in a plane inclined at an angle of 60°.

10. Prove that the sun's image through a convex lens is a circle, and find how the density of rays therein varies.

11. Three persons divide a cylindrical pipe of wine between them, which is emptied through a small orifice in the bottom in 3 quarters of an hour, each taking as his portion what is emptied in 1 quarter of an hour. In what proportion must they pay for it?

12. Explain the principle of the hydrometer, and shew, that if it be made to sink to the same depth in different fluids, the specific gravities of the fluids will be as the weights of the instrument.

13. Find the proportion in which the spaces occupied by the air in the gauge decrease after each turn of the piston in a condenser.

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14. In what part of his orbit is the brightness of Mars the least?

15. Shew where a superior planet is a morning or evening star.

16. Find the solar ecliptic limits.

17. Compare the angular velocity of the distance with the angular velocity of the perpendicular, in general, and in the conic sections.

18. Investigate the expression for the difference of the forces when the movable orbit is an ellipse with the center of force in the center.

19. How must the force of gravity be altered that the times of oscillation in a circular arc may be equal.

20. Find the fluxions of $\overline{a+bx+cx^2}\Big|^{\frac{1}{2}}$, $\frac{1}{\sqrt{a+x}}$, and $\overline{a^x}\Big|^n$.

21. Find the following fluents $\frac{2a\dot{x}}{x\sqrt{a^2+x^2}}$, and $\frac{r^{\frac{1}{2}}x\dot{x}}{\sqrt{r^3-x^3}}$

22. Determine the apparent path of a star's aberration parallel to the ecliptic, supposing the earth to describe any one of the conic sections.

First Afternoon.—Mr. Hornbuckle.

FOR THE FIFTH AND SIXTH CLASSES.

1. Find the chord of curvature, at any point of a parabola, parallel to it axis.

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2. Find the altitude, horizontal range, and time, supposing a body to be projected at a given angle with a given velocity.

3. Prove that the density of the air is proportional to its compressing force.

4. Prove the rule for division of fractions.

5. Find the equation of limits to the general equation $x^n - px^{n-1} + qx^{n-2} - \&c. = 0$.

6. Find the law of force tending to the center of the logarithmic spiral.

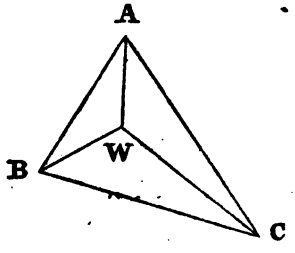
First Evening.—Mr. Sowerby.

1. It is required to find all the possible values of any two numbers X and Y, so that the difference of their squares may be equal to the square of 24.

2. Of all the lines drawn through the focus of an ellipse, and terminated both ways by its perimeter, that is the least which is perpendicular to the major axis. Required a demonstration.

3. It is required to determine that arc of a given circle, whose cosine is equal to its tangent.

4. Let a given weight (W) be supported by three props A,B,C, as in the annexed figure. The pressure upon each prop is proportional to the area of the triangle opposite to it; that is, the pressure on A : pressure on B :: area of the triangle BWC : area of the triangle AWC.



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5. Given the length of a cylindrical beam, having its ends placed upon two planes inclined to the horizon at given angles. It is required to find the position in which the beam will rest.

6. A lever whose arms are inclined to each other at a given angle, and whose lengths and weights are respectively known, is made to vibrate flatways round an axis of suspension which passes through the angular point of the lever. It is required to determine the actual time of an oscillation.

7. A cone of given dimensions is filled with fluid, and placed with its slant side parallel to the horizon. How long will the fluid be in running out of it, through a given orifice in the vertex?

8. The velocity and direction of the wind being known, and also the velocity and direction of a ship in motion; it is required to find the position of the sail with respect to the wind, so that the ship may be impelled with the greatest force possible.

9. Given the difference of the times of setting of two stars whose declinations are known. It is required to determine the latitude of the place.

10. Supposing the latitude and longitude of a star to be known; it is required to determine at what hour it will pass the meridian on that day, when its apparent latitude is neither increased nor diminished by the aberration.

11. In the latitude of 52°, the substyle-line of a vertical dial coincides exactly with the hour-line of 11 o'clock. What is the position of the plane of the dial?

12. A cylindrical vessel of given dimensions is filled with water, and placed with its side perpendi-

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cular to the horizon. At what distance from the vessel must a person stand, so that he may just see the center of the base of the cylinder; the ratio of the sines of incidence and refraction being given; and also the height of the eye above the surface of the fluid?

13. Supposing that the periodic times of two bodies revolving in a given circle are the same, and that the one is acted upon by a force situated in the center, the other by a force situated in the circumference; what is the relation of the absolute forces?

14. What must be the law of the force acting upon a body in a logarithmic spiral, so as to cause it to descend from any point in the curve to the center, always in the same time?

15. Supposing a repulsive force to vary inversely as the cube of the distance from a given plane; it is required to determine the trajectory described by a body projected with a given velocity, and at a given distance from the plane, in the direction of a line parallel to the plane.

16. Supposing the earth and moon to move in circular orbits, and that the radii of their orbits and periodic times are known; it is required to determine whether the moon's orbit, in fixt space, is concave or convex to the sun, when the moon is in conjunction.

17. It is required to determine the law of the resistance (according to the method of NEWTON, Vol. II. Sec. 4) by which a body may be made to revolve in a parabola round a center of force situated in its focus, the force varying as any power of the distance.

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18. In the equation $\frac{p\dot{x}}{x}+\frac{r\dot{y}}{y}=\frac{x^m\dot{x}}{ay^n}$; it is required to find the relation between x and y, when p and r are any numbers whatever.

19. Find the following fluents:

I. $\frac{\dot{x}}{x}.\overline{a^2+x^2}\Big|^{\frac{1}{2}}$.

II. $V^2\dot{x}$, where $V=$hyp. log. $\overline{1+x}$.

III. $Xx^2\dot{x}$, where X is a circular arc whose radius $=1$, and tangent $=\frac{x}{\sqrt{r}}$.

20. Sum the following series:

I. $\frac{1}{1.5}-\frac{1}{2.6}+\frac{1}{3.7}-$&c. to n terms.

II. Given the sum of the series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+$ &c. to find the sum of the series $\frac{1}{1.2^2.3^2}+\frac{1}{2.3^2.4^2}+\frac{1}{3.4^2.5^2}$&c.

21. Required the nature of the curve along which a heavy body, descending by the force of gravity, will press upon the curve at any point, with a force proportional to the ordinate at that point.

22. It is required to determine that point in a parabola, to which a line drawn from the vertex makes the greatest angle with the curve.

23. A ship sails from the equator on a N. W. course. What number of miles will she have run when arrived at the pole, and what will be her difference of longitude?

24. Let AD, AE be two lines given in position; and let the line BC be moved between them so as

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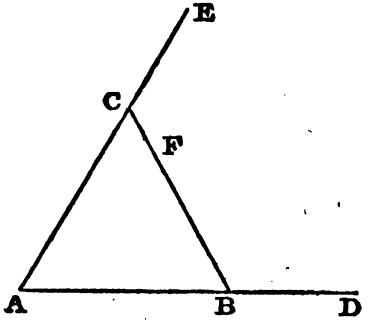
always to cut off an area ABC equal to a given area. It is required to find the nature of the curve generated by a point F, which divides the line BC in any given ratio.



25. Let the roots of the equation $x^n - px^{n-1} + qx^{n-2} -$ &c. $= 0$, be a, b, c, &c. and those of the equation $nx^{n-1} - \overline{n-1}.px^{n-2} + \overline{n-2}.qx^{n-3} -$ &c. $= 0$, be α, β, γ, &c.; then, if when α, β, γ, &c. are successively substituted in the equation $x^n - px^{n-1} + qx^{n-2}$ &c. $= 0$, the results are P, Q, R, &c. and when a, b, c, &c. are substituted in the equation $nx^{n-1} - \overline{n-1}.px^{n-2} - \overline{n-2}.qx^{n-3} -$ &c. $= 0$, the results are p, q, r, &c. it will be as $P \times Q \times R$, &c. : $p \times q \times r$, &c. :: 1 : n^n.

Second Morning.—Mr. Sowerby.

TUESDAY, JANUARY 15, 1805.

1. If a series of arcs be taken in arithmetic progression, the radius of the circle, will be to twice the cosine of their common difference, as the sine of any one arc taken as a mean, to the sum of the sines of any two equidistant extremes. Prove this proposition; and shew how, by means of this property, a table of sines, tangents, &c. may be constructed.

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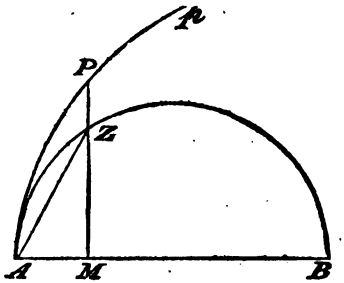
2. Find the sum of n terms of the series $\frac{5.6}{1.2.3.4}$ $+\frac{6.7.}{2.3.4.5}+\frac{7.8.}{3.4.5.6}+$ &c.

3. Suppose a person of given weight to be suspended in a scale from the extremity of an immoveable lever, and to press upwards, by means of a rod of a given length, against the under-side of the lever; with what force must he press upwards, so that he may rest in any given position?

4. The curve APp is generated by taking its ordinate MP always equal to the corresponding chord of the circular arc AZ. Required its nature, and also its area, supposing it to terminate when its abscissa AM becomes equal to the diameter AB of the circle.



5. If a sphere and cylinder of the same diameter move with equal velocities in the same fluid, in the direction of the cylinder's axis, the resistance opposed to the motion of the globe will be to the resistance opposed to the cylinder in the ratio of one to two. Required a demonstration.

6. Given the greatest and least horizontal parallaxes of the moon. It is required to find her mean distance in terms of the radius of the earth.

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7. To determine the *nature* and *length* of the caustic, when the reflecting curve is a circular arc, and the focus of incident rays is in the circumference of the circle.

8. A body is projected, at a given distance from a centre of force, with a velocity and direction which will cause it to move in the reciprocal spiral, the force varying inversely as the cube of the distance. It is required to investigate Cotes's construction for determining the place of the body at the end of any given time.

9. Given the densities of the earth and Jupiter, the times of their diurnal revolutions, and the polar and equatorial diameters of the earth, to find the ratio between the polar and equatorial diameters of Jupiter.

10. Explain the CARTESIAN hypothesis of vertices, and shew that it will not satisfactorily account for the phænomena of the heavenly bodies.

11. Given the latitudes and longitudes of two places upon the surface of the earth. It is required to determine their distance from each other upon MERCATOR'S chart.

12. Supposing the sections of a groin, made by a plane passing through its axis, and cutting the opposite sides of the base at right angles, to be circles; what will be the nature of the section when the plane cuts the sides of the base at any other angle?

13. Given the latitude of the place, the declination of the sun, and the position of a plane, both with respect to the meridian, and the horizon. It is required to find at what hour of the day the sun will begin to shine upon it.

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14. If the cover of a common lamp in the streets be a perfect circle, whose plane is perpendicular to the wall to which the lamp is attached; what will be the nature of its shadow on the wall, supposing the wick to be a point situate in the axis of the cover?

Second Afternoon.—Mr. Sowerby.

FOR THE THIRD, FOURTH, FIFTH, AND SIXTH CLASSES.

1. Find the vulgar fraction which is equivalent to the circulating decimal ,7485353.

2. In the extraction of the square roots of numbers, it is required to investigate the limit which the remainder after any operation can never exceed.

3. Find the roots of the equation $x^3+\frac{10}{7}x^2-\frac{4000}{9261}=0$, two of them being equal.

4. A weight *(P)* draws another weight *(W)* along an inclined plane given in position, by means of a rope passing over a fixt pully. It is required to find the position of the weight when in equilibrio.

5. In what direction must a ray issuing from a given point be incident upon a given plane refracting surface, so that after refraction it may converge to another given point in the axis of the pencil.

6. In determining the velocity of a fluid issuing from an orifice in the bottom of a vessel, some writers have found it to be that which a heavy body would acquire in falling through the whole height of the fluid in the vessel; others, that acquired in falling through half the height. How have these different conclusions been obtained, and which is the true one?

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7. Suppose a person suspended in a balance to act upwards, by means of a rod against a point in the arm of the lever opposite to that in which he is suspended: will his weight be increased or diminished by this action, and in what ratio?

8. Supposing that in the GREGORIAN telescope the focal lengths of the reflectors are given, and also the distance of the last image from the principal focus of the large reflector. It is required to determine the distance of the reflectors from each other.

9. Explain the phænomenon of the harvest moon.

10. Given the difference of the lengths of the shadow of a lofty tower observed at mid-day, in the summer and winter solstices, and also the latitude of the place. It is required to determine the height of the tower.

11. In any conic section, the centripetal is to the centrifugal force as the distance of the body from the focus to half the principal latus rectum of the figure.

12. How far must a body fall *internally*, and how far *externally*, to acquire the velocity which is necessary to retain it in a circle, supposing it in both cases to be acted upon by a force which varies inversely as the square of the distance?

13. Supposing the force in any orbit nearly circular to be represented at any point by the quantity $\frac{bA^m+cA^n}{A^3}$, it is required to determine the angle between the apsides.

14. Find the sum of n terms of the series, $1.2+2.3+3.4+4.5+$&c.

15. To place a given straight line in a circle of

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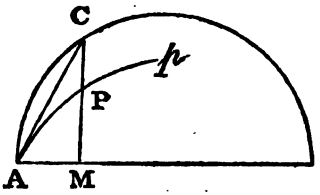
given magnitude, so that it may pass through a given point, either within or without the circle.

16. Find the fluent of $\frac{x^2\dot{x}}{\sqrt{a^2+x^2}}$, and also that of $\frac{\dot{z}}{1+2az+z^2}$, where a is less than unity.

17. Let the curve APp be generated by taking the ordinate MP always equal to the difference of the chord AC, and the versed sine AM of the circular arc AC. It is required to determine the greatest ordinate of the curve, and also its area.



18. If any momenta be communicated to the parts of a system, its center of gravity will move in the same manner that a body equal to the sum of the bodies in the system would move, were it placed in that center, and the same momenta communicated to it, in the same directions.

19. Let the equation to a curve be $xy^2=a^3$, where x is the abscissa and y the ordinate. It is required to determine the area comprehended between any two ordinates, b and c.

20. The resistance opposed to a plane moving perpendicularly in a fluid with a given velocity, is to the resistance opposed to its motion when moving obliquely, as the cube of the radius to the cube of the sine of the inclination of the plane to the direction of its motion.

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Second Afternoon.—Mr. Sowerby.

1. Investigate a rule for extracting the square root of a binomial, one or both of whose factors are quadratic surds; and apply it to determine the square root of $2\sqrt{-1}$.

2. The spaces described in any times by bodies uniformly accelerated, are proportional to the squares of the times, or to the squares of the last acquired velocities.

3. Explain the principle and construction of the air-pump; and compare the density of the air in the receiver at first, with its density after any number of turns.

4. Find the principal focus of a sphere, the radius of the sphere and the ratio of the sines of incidence and refraction being given.

5. Suppose the body to revolve in an ellipse, it is required to find the law of the force tending to the focus.

6. Divide a given angle into two other angles, so that their sines may be to each other in a given ratio; and shew how the value of these angles may be calculated.

Second Evening.—Mr. Hornbuckle.

1. In a given latitude, a pendulum will oscillate once in one second, supposing the earth not to revolve round its axis. Required the angular motion round the axis, that the pendulum may oscillate once in two seconds.

2. $a^x . b^y . c^z$ is a minimum, and $\sqrt{x+1} . \sqrt{y+1} . \sqrt{z+1} = 2$ a constant quantity. Required the relation between x, y, z.

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3. Resolve $\frac{1}{1-cz+fz^2-gz^3+\&c.}$ into a series of fractions whose denominators are binomials.

4. Determine the apparent magnitude of a straight rectilinear object, placed at a given depth, parallel to a surface of water; the eye being situated at any point in the plane passing through the object perpendicular to the surface.

5. Two perpendiculars of given lengths are situated at a given distance from each other in a horizontal plane. Determine geometrically that point on the plane between them, to which, if lines be drawn from the extremity of each perpendicular, the times of falling down the two inclined planes may be equal.

6. Find the center of gravity of a cylindrical portion of the atmosphere measured from the earth's surface, the force of gravity being constant.

7. If the ordinate (y) of a curve be composed of powers of the abscissa x and constant quantities; having given the increment of x, find the contemporary increment of y.

8. A string of given length is suspended to two tacks any where situated, the length of the string being greater than the distance between them. It is required to find the position of a given weight (w), which slides freely on the string, when at rest.

9. Construct the spiral in which the areas are the measures of the ratios between the ordinates which terminate them.

10. The angle between the apsides in an orbit very nearly circular : $180^\circ :: \sqrt{b+c} : \sqrt{mb+nc}$. Prove that the law of the force hence deduced by NEWTON'S method coincides with $\frac{bA^m+cA^n}{A^3}$.

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11. Required the time in which a given cylindrical wheel will roll from the top of a given conical hill to the bottom.

12. Required the present value of £1. to be paid at the end of n years, if either of the individuals A or B, whose ages are given, be alive at that time.

13. It is required to find the altitude of the first point of Aries, at a given hour, day, and place; the angle also, and point in which the ecliptic cuts the horizon at that time.

14. Given the length and weight of an elastic string, and the force which stretches it, to find the number of vibrations in a second.

15. Construct the fluent $\frac{\dot{z}}{z+2az^2+z^3}$, a being less than unity.

16. A reservoir being supplied with water at a given rate, determine the height to which a sluice must be drawn, that the reservoir may be always kept just full; the dimensions of the sluice, and the depth of its base from the surface, being given.

17. Explain the reason why at spring-tides in summer, in north latitudes, the afternoon tide is greater than the morning tide.

18. A particle is attracted towards a straight line given in position and magnitude, the law of the force being the inverse square of the distance. Determine the direction in which the particle will begin to move towards the line.

19. Find the fluents of the following quantities $\dot{x} \cdot \int \dot{x} \cdot \int \dot{x} \cdot \int \dot{x}$; $x^a \cdot a^x \dot{x}$ (a being a given quantity;) $\frac{\dot{x}}{\overline{x-a}^2 \cdot \overline{x-b}^3}$.

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20. Find the integral of the increment $\frac{1}{V^{\frac{2}{5}}}$, V being one.

21. The roots of the quadratic equation $x^2 - px + 1 = 0$ are a and b. Prove that $a^n + b^n$ is equal to

$$p^n - np^{n-2} + n \cdot \frac{n-3}{2} p^{n-4} - n \cdot \frac{n-4}{2} \cdot \frac{n-5}{3} p^{n-6} + \&c.$$

22. BRADLEY observed, that the apparent motion in declination of every star tended the same way when they passed the meridian at the same hour, each being farthest north when it passed at six o'clock in the evening, and farthest south at six in the morning. Explain the reason why this is nearly true in those stars whose declinations are not very great.

23. A ball is shot from a cannon with a given velocity, at a given angle of elevation, situated at a given distance from the foot of a hill, whose elevation is also known. Determine the point in which the ball will strike the hill.

24. Given the force of gravity at the surface of a primary planet, the mean distance and periodic time of its secondary with the ratio of their respective diameters. It is required to compare their densities.

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1806.

First Morning.—Mr. J. Brown.

MONDAY, JANUARY 13, 1806.

1. How many years' purchase is an estate worth, when the rate of interest is 4 per cent.?

2. An inclined plane is a tangent to a cycloid in the middle point between the highest and lowest points; what must be the length of the plane, that the time of falling down it may be equal to the time of half an oscillation?

3. Having given the specific gravities of water and iron, it is required to determine what proportion the thickness of an hollow iron globe must bear to its diameter, that it may just float in water.

4. In a given latitude, and at a given time of the year, how many hours will be shewn upon a vertical south dial? And at what time of the year will the greatest number be shewn?

5. Having given the altitudes of the sun, and of any particular colour in the primary rainbow, it is required to determine the sines of incidence and refraction.

6. The curve ANC is a cycloid, and the curve AMD is formed by taking PM equal to the arc

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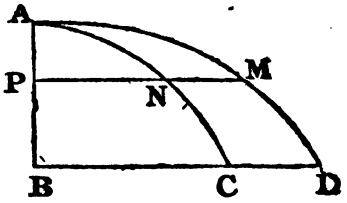
AN; find the equation to the curve, and compare its area with that of the cycloid.



7. Compare the attractions of a spheroid upon a particle on its surface in directions parallel to the polar and equatorial diameters, with the attractions at the pole and the equator.

8. A paraboloid with its vertex downwards is emptied by an orifice in its vertex; compare the times in which the fluid descends through the first and last half of the axis.

9. Draw a diameter to the curve, whose equation is $y^n - \overline{a+bx}\,.y^{n-1} + \&c. = 0$.

10. Sum the following series:

$1^3 + 2^3 + 3^3 + \&c.$ to n terms.

$\frac{1}{1.3} + \frac{1}{2.4} + \frac{1}{3.5}$ &c. in inf.

$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \&c.$ in inf.

11. Find the following fluents: $\frac{x\dot{x}}{\sqrt{2ax-x^2}}$, $\frac{\dot{x}}{1-x^4}$, $\frac{z\dot{z}}{1+2az+z^2}$, where a is less than unity; and construct the latter.

12. Trace the curve whose equation is $\overline{a^2-x^2} + \overline{x-b}|^2 = x^2y^2$.

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13. Shew that the altitude of high water above the mean altitude is equal to twice the depression below it.

14. Find the radius of curvature of the common parabola, and the equation to the evolute.

15. Shew that if the resistance in any medium is proportional to the velocity, the oscillations in a cycloid will be isochronous.

16. Supposing the moon to revolve round the earth in a circle, what must be the diminution of the quantity of matter in the earth, that the eccentricity of her orbit may be equal to half the radius of her present orbit?

First Afternoon.—Mr. J. Brown.

FOR THE THIRD, FOURTH, FIFTH, AND SIXTH CLASSES.

1. Find the value of .78545454 &c. in a vulgar fraction.

2. If £40. be due at the end of 6 months, £60. at the end of 12 months, and £80. at the end of 15 months; What is the equated time for paying the whole?

3. The sum of four numbers in arithmetical progression is 56, and the sum of their squares is 864; What are the numbers?

4. If the latitude of the place, and the sun's declination be given; it is required to find the time of his rising, his azimuth at that time, and the time of his coming to the prime vertical.

5. A body has fallen from A to B, when another body is let fall from C; How far will

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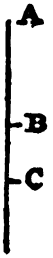
the latter body fall before it is overtaken by the former.



6. If R and r be the radii of the surfaces of a convex lens, and m and n the sines of incidence and refraction; Find the focal length of the lens, and shew that it is the same which ever way the rays pass.

7. Having given the three angles of a spherical triangle, and the radius of the sphere, find the area of the triangle.

8. Find the ecliptic limits of the sun and moon, and how many eclipses there may be in one year.

9. Inscribe the greatest cone in a given sphere.

10. Find how far a body must fall internally to acquire the velocity in a circle, when the force varies inversely as the square of the distance.

11. When a body revolves in any curve, compare the angular velocity of the distance with that of a perpendicular upon the tangent.

12. Shew that the periodic times in all ellipses round the same centre are equal.

13. If the force varies inversely as the cube of the distance, and the whole distance be made the radius of a circle; then, if the versed sine of any angle represent the space fallen through, the right

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sine will represent the time, and the tangent the velocity acquired.—Required a proof.

14. Solve the following cubic equation, whose roots are in geometric progression: $x^3 - 13x^2 + 39x - 27 = 0$.

15. Shew that the time of an oscillation in a cycloid : time down the axis :: the circumference of a circle : the diameter.—And having given the length of a pendulum, which vibrates seconds, find how far a body will fall by the force of gravity in one second.

16. Compare the times of emptying a cone and its circumscribing cylinder, through equal orifices in the base of the cylinder, and the vertex of the cone.

17. Shew that if no light were intercepted in passing through the air, an object would appear equally bright at all distances.

18. A barometrical tube is 36 inches long, and after inversion the mercury stands at the height of 24 inches; What quantity of air was left in the tube before inversion, supposing the standard height to be 30 inches?

19. Find the following fluents: $\frac{2a\dot{x}}{a^2 - x^2}$, $\frac{x^2\dot{x}}{\sqrt{a^2 + x^2}}$, and $\frac{\dot{z}}{1 + 2az + z^2}$, where a is less than unity.

20. Sum the following series:

$$\frac{3}{1.2.2} + \frac{4}{2.3.2^2} + \frac{5}{3.4.2^3} + \text{\&c. in inf.}$$

$1.2 + 2.3 + 3.4 + 4.5$ &c. to n terms.

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21. If a, b, c, &c. be the roots of the equation $x^n - px^{n-1} + qx^{n-2} -$ &c. $= 0$; it is required to transform it into an equation, whose roots are ma, mb, mc, &c.

22. Shew that the attractions of similar pyramids, whether whole or obtruncated, upon particles situated in their vertices, are proportional to their lengths.

23. At what angle must a body be projected with a given velocity, that the area of the parabola described may be a maximum.

24. Determine the angle between the apsides when the force varies as $\frac{bA^m + cA^n}{A^3}$.

First Evening.—Mr. Barnes.

1. The sum of the cosines of any two arcs, is to their difference, as the co-tangent of half their sum, to the tangent of half their difference.

2. Given the latitude of the place, and the right ascension and declination of a star, to find the hour at which it will rise on a given day.

3. Find the focal length of a thin concavo-convex lens of glass, the radii of the surfaces being given.

4. Construct and investigate the magnifying power of the double microscope.

5. The roots of the equation $4x^3 - 32x^2 - x + 8 = 0$ are of the form a, $-a$, b. It is required to determine them.

6. If a be an integral root of the equation $x^n + px^{n-1} +$ &c. $\ldots\ldots + Px^3 + Qx^2 + Rx + S = 0$, and if $\frac{S}{a} + R = R'$; $\frac{R'}{a} + Q = Q'$; $\frac{Q'}{a} + P = P'$ &c.;

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then there are $\frac{S}{a}$, R, $2'$, P' &c. integers. Required a demonstration.

7. Given the specific gravities of two bodies, to find the ratio of their quantities of matter, when they balance each other in a fluid whose specific gravity is also given.

8. A given cylindric vessel is supplied with water by a cock at a given rate. Then suppose when the vessel is full, that an aperture of given area is made in its bottom; what is the lowest point to which the surface of the water in the vessel will descend, and also the time of its descent; the influx of the water by the cock being supposed slower than the efflux when the vessel is full?

9. A given weight, suspended by a thread of a given length, oscillates, in the manner of a pendulum, through a semi-circle. Compare the weight of the body with the force which stretches the string in any position.

10. Find the magnitude of that part of the disturbing force of the sun upon the moon, which acts in a direction perpendicular to the plane of the moon's orbit.

11. Required the fluents of $x^3\dot{x} \times \overline{a^2+x^2}^{\frac{1}{2}}$, $\frac{d \times \overline{z}^{\frac{1}{2}n-1}\dot{z}}{a+bz^n\sqrt{c+ez^n}}$ and $\frac{\dot{z}}{z^n \times a+bz}$.

12. Find the relation of x to y in the equation $x\dot{y} - y\dot{x} = \dot{x}\sqrt{x^2+y^2}$.

13. Required the sum of n terms of the series $1.2.4+2.3.5+3.4.6+$ &c.; and the sum of the series $1+\frac{1}{3.5}+\frac{1}{5.9}+\frac{1}{7.13}+$ &c. ad inf.

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14. Given the sum of the series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+$ &c. equal to $\frac{2\mathcal{Q}^2}{3}$, ($\mathcal{Q}$ being the length of the quadrantal arc to the radius $=1$); to find the sum of the series $\frac{1}{1^2.2}+\frac{1}{2^2.3}+\frac{1}{3^2.4}+$ &c. ad inf.

15. The fluent of $y\dot{x}=y\times\frac{x}{1}-\frac{\dot{y}}{\dot{x}}\times\frac{x^2}{1.2}+\frac{\ddot{y}}{\dot{x}^2}\times\frac{x^3}{1.2.3}-$ &c. ($\dot{x}$ being constant); required the investigation.

16 At what distance from its extremities must a slender rod, of uniform thickness and given weight, be suspended, that it may oscillate in a given time?

17. Suppose the proportion of the chances for the happening of an event, be to that of its failing in any one trial as a to b; what is the probability that the event will happen precisely p times in n trials?

18. If z be constant, $\frac{1}{z}=\frac{1}{z}+\frac{nz}{zz}+\frac{n.n+1.z^2}{zzz}+$ &c. required the investigation.

19. If a body revolve in an ellipse round a center of force situated in the focus; investigate the relation between the mean angular velocity of the body, and its angular velocity round the upper focus; and shew from this relation that when the ellipse is of small excentricity, the angular velocity round the upper focus is nearly uniform.

20. The greatest velocity which can be acquired by a spherical body descending in a fluid, is equal to

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that which would be acquired by it in descending from rest in vacuo, by the constant force of its comparative gravity, through a space which is to $\frac{4}{3}$ of the diameter, as the density of the sphere to the density of the fluid.

21. Suppose the force of gravity to be uniform, and to tend perpendicular to the horizon; what will be the path of a projectile in a medium, the resistance of which is proportional to the velocity of the body?

22. If the centripetal force vary reciprocally as the distance from the centre; and the density of the fluid be proportional to the compressing force; then if distances from the centre be taken in geometrical progression, the corresponding densities of the fluid will be also in geometrical progression.

23. Bradley observed that every star passed the meridian farthest south when it came about six in the morning, whatever were its position with respect to the cardinal points of the ecliptic. Suppose then the place of a star to be given; on what day will it pass the meridian of London farthest to the south; and at what hour will it pass it on that day?

24. A perfectly flexible chain is wound round a cylinder supported with its axis parallel to the horizon. Then, if the weight and dimensions of the cylinder be given, and also the length and weight of the chain; it is required to determine the time in which the chain, impelled by the force of gravity, will unwind itself; a given length being unwound at the commencement of the motion.

Second Morning.—Mr. Barnes.

1. Find the value of the decimal ,726363 &c. in a vulgar fraction.

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2. Required the value of x in the equation $2x\sqrt[3]{x} - 3x\sqrt[3]{\frac{1}{x}} = 20$.

3. If $x^n - px^{n-1} + qx^{n-2} - \&c. \pm W = A$, and a be any root of the equation $x^n - px^{n-1} + qx^{n-2} - \&c. \pm W = 0$; Prove that $x - a$ is a divisor of the expression $x^n - px^{n-1} + qx^{n-2} - \&c. \pm W$.

4. Suppose the arms of a bent lever, inclined to each other at a given angle at the fulcrum, to be also given: Required the position of the lever, when two given weights, suspended freely from its extremities, balance each other.

5. A given orifice is opened in the bottom of a vessel of given altitude filled with water; What must be the nature and dimensions of the vessel, that the surface of the water may descend with a given uniform velocity?

6. Required the nature of the caustic, when the reflecting curve is the common cycloid, and the rays are incident parallel to the axis.

7. Two stars whose right ascensions and declinations are known, were observed to rise at the same moment: Required the latitude of the place of observation.

8. Prove that the periodic times of bodies, describing different ellipses round different centres of force situated in their foci, are in the sesquiplicate ratio of the major axes directly, and in the sub-duplicate ratio of the absolute forces inversely.

9. Find the fluents of $\frac{x^3\dot{x}}{\sqrt{a^2 - x^2}}$ and of $\frac{\dot{x}}{x^3\sqrt{a^2 - x^2}}$.

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10. Required the sum of the first n terms of the series $\frac{1}{1.3}-\frac{1}{2.4}+\frac{1}{3.5}-\frac{1}{4.6}$ &c. n being an even number.

11. Find the sum of the first n terms of the series $\frac{1}{1.2.4}+\frac{1}{2.3.5}+\frac{1}{3.4.6}+$ &c. by the method of increments.

12. Compare the mean quantity of the force by which the sun disturbs the motion of the moon, with the force of gravity at the earth's surface.

13. Compare the force which accelerates the centre of gravity of a cylinder, while it rolls down a given inclined plane, with the weight of the cylinder.

14. Given the latitude of the place, and the right ascension and declination of a star, to determine the time of the year when the star rises with the sun.

Second Afternoon.—Mr. Barnes.

FOR THE THIRD, FOURTH, FIFTH, AND SIXTH CLASSES.

1. What is the interest of £400. for 9 months at $4\frac{1}{2}$ per cent. per annum?

2. What is the discount of the same sum for the same time, at the same rate of interest?

3. Reduce 1.6363 &c. of a crown to the fraction of a guinea.

4. Given $3x^2-3x+6=5\frac{1}{3}$, to find x.

5. Required the sum of n terms of the series, $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-$ &c.; and also the sum of the series continued ad infinitum.

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6. Find the sum of n terms of the series a, $a+b$, $a+2b$, $a+3b$, &c.

7. With what velocity must a body be projected from a given point, in a given direction, to hit a given mark?

8. Given the specific gravity of a fluid, and also of a solid in the form of a paraboloid floating in it with its vertex downwards, to find the part of the axis immersed.

9. Given the distance of the object from a double convex lens, and the ratio of the object to the image, to find the focal length of the lens.

10. If a body revolve in an ellipse round a centre of force situated in one of its foci; prove that the velocity in any point, is to that in a circle at the same distance, in the sub-duplicate ratio of the distance from the other focus, to half the major axis.

11. Given the latitude of the place, and the declination of the sun, to find the length of the day.

12. If the signs of the alternate terms of an equation beginning with the second, be changed, the signs of all the roots are changed.

13. Required the fluxions of $\frac{a}{\sqrt{x^2+y}}$, and of $\frac{\overline{x+a}^2}{\sqrt{x^2-a^2}}$

14. Find the fluents of $\frac{bx\dot{x}}{\sqrt{a+x}}$, and of $\frac{2ax^2\dot{x}}{a^2-x^2}$.

15. If the force vary directly as the distance from the centre, shew that the velocity of a falling body, is to that in a circle at the same distance, as the right sine to the cosine of a circular arc, whose radius is the

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greatest distance, and versed sine the space through which the body has fallen.

16. At what season of the year is that part of the equation of time, which arises from the inclination of the ecliptic to the equinoctial, a maximum?

17. If two imperfectly elastic balls strike each other, prove that the relative velocity before impact, is to the relative velocity after, as perfect to imperfect elasticity.

18. If perpendiculars be drawn from any number of bodies to a given plane, the sum of the products of each body multiplied by its perpendicular distance from the plane, is equal to the product of all the bodies multiplied by the perpendicular distance of their common centre of gravity from the plane.

19. Investigate the fluxional expression for the area of any curve; and apply it to find the area of the curve whose equation is $a^m x^n = y^{n+m}$ comprehended between the ordinates, whose lengths are b and c.

20. Find that point in the moon's orbit (supposing it a circle,) where the disturbing force of the sun neither increases nor diminishes the gravitation of the moon towards the earth.

Second Afternoon.—Mr. Barnes.

FIFTH AND SIXTH CLASSES.

1. Reduce $\frac{5}{7}$ of a guinea to the decimal of a pound.

2. Prove the rule for finding the greatest common measure of two numbers.

3. Reduce the fraction $\frac{209}{380}$ to its lowest terms.

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4. Required the value of x in the equation $2x^4 - x^2 + 104 = 600$.

5. Prove that if a body be projected in any direction inclined to the horizon, it will describe a parabola.

6. Find the law of the force by which a body is made to revolve in a parabola, round a centre of force situated in the focus.

7. Shew that the pressure of the fluid upon any surface immersed in it, is equal to the weight of a cylindric column of the fluid, whose base is the surface pressed, and altitude the perpendicular depth of the centre of gravity of the surface pressed below the surface of the fluid.

8. Construct and investigate the magnifying power of the common astronomical telescope.

Second Evening.—Mr. J. Brown.

FOR THE FIRST, SECOND, THIRD, AND FOURTH CLASSES.

1. A pendulum, which kept true time at the earth's surface, when carried to the top of a mountain lost n seconds a day; What was the height of the mountain?

2. Having given the places of the sun and moon at any time, it is required to delineate the phases of the moon, and to find the angle which her cusps make with the horizon.

3. Shew in what positions of the eye and object, the image formed by a double convex lens will be magnified or diminished; and when it will be erect, and when inverted.

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4. Having given the weight and magnitude of a balloon, and the specific gravities of atmospheric air, and of the air with which the balloon is filled, find the height to which it will ascend.

5. Shew how an eclipse of the moon, with the time of its duration, and the number of digits eclipsed, may be calculated.

6. Compare the times of emptying a globe and its circumscribing cylinder through equal orifices; the orifice of the cylinder being in the base.

7. A given weight P is connected with a cylinder by means of a string wound round it, and descends 48 feet in 2 seconds, causing the cylinder to revolve round its axis; What is the weight of the cylinder?

8. A cylindrical vessel, whose height is equal to its diameter, is filled with water; With what velocity must it be whirled round its axis, that half the water may be thrown out?

9. Approximate to the root of the equation $x^3+x^2+x=90$; and shew that the accuracy of the approximation depends upon the quantity assumed being much nearer to one root than to any other.

10. If the earth and moon were at rest, having given their quantities of matter and densities, it is required to determine the least velocity, with which a body projected from the moon would fall to the earth.

11. If a body be projected from an apse with the velocity acquired in falling from an infinite distance when the force varies inversely as the n^{th} power of the distance; in how many revolutions, or in what parts of a revolution, will it either fall into the centre, or go off to infinity?

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12. When a globe oscillates in a resisting medium, having given the diameters of the globe and of the string, compare the resistance upon the string with the resistance upon the globe.

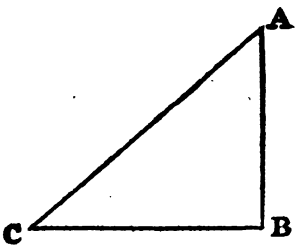
13. Divide a given angle into two such parts, that the cube of the sine of one part multiplied into the square of the sine of the other may be a maximum. And give the geometrical construction for dividing the angle.

14. Resolve the fluxional equation $x\dot{x}+ay\dot{x}-y\dot{y}=1$.

15. Supposing the moon's orbit nearly circular, find the increase of her velocity by the tangential force, while she moves from quadrature to syzygy.

16. Resolve $\frac{1}{1+x^n}$ into its quadratic divisors.

17. The height of the tower AB is to be determined by measuring the base BC, and taking the angle ACB; supposing there to be some small error in the observation, at what distance from the foot of the tower should it be made, that the corresponding error in the height of the tower may be a minimum?



18. Having given the fluent of $\overline{a+cz^n}^m \times z^{pn-1}\dot{z}$, find the fluent of $\overline{a+cz^n}^{m+r} \times z^{pn+vn-1}\dot{z}$.

Find the fluent of $v^2\dot{x}$, where $v=$ the hyp. log. of $x+\sqrt{a^2+x^2}$.

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Find the fluent of $\frac{\dot{x}}{\sqrt{1+x^2}}$.

19. Find the magnifying power, and field view, of Cassegrain's telescope.

20. Sum the following series:

$\frac{1}{1.5}+\frac{1}{3.7}+\frac{1}{5.9}$ to n terms, and ad. inf.

$\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+\frac{1}{7.8}$ &c. ad. inf.

$\frac{1}{1.2.5}-\frac{1}{2.3.7}+\frac{1}{3.4.9}-$ &c. ad. inf.

$1-\frac{1}{3.3}+\frac{1}{5.3^2}-\frac{1}{7.3^3}+$ &c. ad inf.

21. Having given the diameter of a primary planet, and the force of gravity upon its surface, and also the periodic time and distance of the secondary revolving in a circle, to find the quantity of matter in the secondary.

22. Explain the principle of interpolations.—And having given on the 19$^{\text{h}}$ of any month, at noon, the sun's declination $=28'.41''$ N.; on the $20^{\text{th}}=5'.0''$ N.; and on the $21^{\text{st}}=18'.41''$ S.; find the time of equinox.

23. Shew that if a globe of fluid matter revolve round its axis, it will put on the form of a spheroid.

24. Shew that every section of a solid formed by the revolution of a conic section round its principal axis is also a conic section.—And if a paraboloid be cut by a plane parallel to its axis, the section will be a parabola similar and equal to the original one.

25. Shew that the area of the least polygon circumscribing a circle: the area of the greatest polygon

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of the same number of sides, inscribed in the circle :: $\overline{\text{radius}}^2$: $\overline{\text{cos.}}^2$ of half the angle subtended by any side.

Mr. J. Brown.—For the Fifth and Sixth Classes.

1. The rectangle contained by the diagonals of a quadrilateral figure inscribed in a circle, is equal to both the rectangles contained by its opposite sides.

2. In the collision of two perfectly elastic bodies, the sum of the products of each body multiplied into the square of its velocity, is the same before and after impact.

3. Shew that, when bodies revolve in circles having the force in centre, the force varies as the radius directly, and the square of the periodic time inversely.

4. Given the sines and cosines of two angles, to find the sines and cosines of their sum and difference.

5. Shew that an arithmetic mean is greater than a geometric one.

6. Sum the following series:

$$1 + \frac{4}{3} + \frac{4}{9} \text{ \&c. in inf.}$$

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} \text{ \&c. in inf.}$$

7. Explain the cause of the change of seasons, and of the difference of the lengths of day and night.

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1807.

First Morning.—Mr. D'Oyly.

MONDAY, JANUARY 19, 1807.

1. Find the least whole number, which, when divided by 3 and 5, has its respective remainders 1 and 3.

2. A spherical triangle has its angles 160°, 150°, and 140°. Compare its area with that of a great circle of the same sphere.

3. The equation $x^3+8x^2+20x+16=0$ has two equal roots. Find the three roots of the equation.

4. The sun's right ascension is 40°, and the latitude of the place 52°. Find the ascensional difference.

5. If the force varies as the cube of the distance, how far must a body fall internally to acquire one-third of the velocity in a circle?

6. A lighter fluid rests on a heavier: their specific gravities are respectively 3 and 6. A paraboloid whose equation is $ax=y^4$, floats between them with its vertex downwards, and sinks with one-fourth of its axis in the heavier. Find the specific gravity of the solid.

7. A groin is generated by a square moving parallel to itself, the section through its opposite sides being a semi-cubical parabola. Find the time in which it empties itself through a given hole in the vertex.

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8. The periphery of a circle revolves on a diameter for its axis. At what distance from this axis is its centre of gyration?

9. Two telescopes, of Sir I. NEWTON's construction, with equal reflectors, have the same degree of indistinctness. Compare their magnifying powers, and their degrees of brightness.

10. Find the relation of x to y in the equation $a\dot{x}^2 = y\dot{y}^2 - b\dot{x}\dot{y}$.

11. There are 4 individuals, of such an age, that, according to the tables of mortality, it is just an even chance whether each will be alive at the end of a year. What is the probability that two of them will die in that time?

12. Sum the series $\frac{1}{3} + \frac{1.2}{3.4} + \frac{1.2.3}{3.4.5} + \frac{1.2.3.4}{3.4.5.6}$ + &c. ad infin.

also, $\frac{1}{1.5} + \frac{1}{3.7} + \frac{1}{5.9}$ + &c. to n terms.

13. Find fluents of $\frac{x\dot{x}}{x-a}$, $\frac{x\dot{x}\sqrt{1-x^2}}{\sqrt{1+x^2}}$, and $\frac{\dot{x}}{\sqrt{1-e^{-mx}}}$, where e is the number whose h. l. is 1.

First Afternoon.—Mr. Woodhouse.

THIRD AND FOURTH CLASSES.

1. Find the interest on 12s. 4d. at $4\frac{1}{2}$ per cent.

2. Given the first term $= 1$, the number of terms $= n$, and the sum $= S$; Find the common difference, the series being arithmetical.

3. Prove that sin. $(60 + A) =$ sin. $(60 - A)$ + sin. A.

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4. Prove that

$$\frac{\sin.a+\sin.b}{\cos.a+\cos.b}=\tan.\left(\frac{a+b}{2}\right)\ (\text{radius}=1.)$$

5. Compare the sun's apparent diameter at the extremity of the latus rectum of the earth's orbit, with the perigean apparent diameter.

6. If two bodies are moved at the same time towards each other, from the two extremities of a vertical line (L); one projected upwards with a velocity acquired down $\frac{3L}{2}$, the other let fall from rest; It is required to find the point where they meet.

7. A cubical vessel is filled with water, and into its side a bent tube is inserted, filled with water, and communicating with the water in the vessel; Required the pressure on the top of the vessel, the vertical height of the extremity of the tube above the vessel being m times the height of the vessel.

8. Let the focus of incidence 2 be $\frac{1}{4}$ radius from the vertex of a spherical concave reflector; Find the place of q, the focus of reflected rays.

9. Let a weight P, descending vertically, draw 2 up an inclined plane, the elevation of which is 30 degrees; Find the velocity of P, after that a time t is elapsed.

10. In NEWT. Prop. 8. Sect. 2. a semi-circle is described by P; what must be the change of circumstances that an ellipse may be described?

11. Find the fluxion of the hyp. log. $\left(\frac{1-x^{\frac{1}{2}}}{1+x^{\frac{1}{2}}}\right)$, and fluent of $x\dot{x}\surd(1+x)$.

12. Shew by what method the obliquity of the ecliptic is determined.

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13. In elastic bodies, shew that, if A overtake B, the velocity of B after impact is greater than the velocity of A.

14. Given $\begin{cases} \text{Latitude} = l, \\ \text{Sun's altitude} = a, \\ \text{Comp. declination} = p: \end{cases}$

It is required to prove, that, t being time from noon,

ver. sin. $t = 2$ cos. $\left(\frac{p+l+a}{2}\right)$ sin. $\left(\frac{p+l+a}{2} - a\right)$ $\times$ cosec. p. sec. l.

15. If a point be kept at rest by any number of forces represented by the sides of a polygon, then any one force is equal and opposite to the result of all the other forces. Required proof.

16. Sum $1+3+6+10+15$ &c. to m terms.

17. Let a body be projected from the top of a tower horizontally, with a velocity acquired in falling down a space equal to the height of the tower. At what distance from the base of the tower will it strike the horizon?

18. Sum $1^3+2^3+3^3+$ &c. $\ldots\ldots n^3$.

by the method of increments.

19. In the equation $x^3 - \frac{7}{2}x^2 + \frac{7}{2}x - 1 = 0$,

two roots are of the form a and $\frac{1}{a}$; It is required to solve the equation.

20. Compare the velocity of a body moving in a parabola, at the vertex, with the velocity at the extremity of the latus rectum; force being in the focus.

21. Let a body revolve in an hyperbola; Required the law of the centrifugal force directed from the center of the hyperbola.

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Fifth and Sixth Classes.—Mr. Woodhouse.

1. What is the interest of £1. 1s. 6d. for one year, at 5 per cent.?

2. Find 4 numbers in arithmetical progression, the common difference being 3, and their product 280.

3. Prove, that tan. $2A$ is greater than 2 tan. A, $A < 45°$.

4. Let 3 equal bodies be placed in the 3 angles of a triangle; Find their centre of gravity.

5. Compare the velocity of a body at the extremity of the latus rectum of an ellipse, with the velocity at the mean distance; force being in the focus.

6. The specific gravities of mercury and water being as 13568 to 1000; What ought to be the length of a water barometer inclined to the horizon at an angle of 30°, the mercury standing at 30. 5 inches?

7. In north latitude 52°. 12′. 36″, and with 1°. 30′. 0″, north declination, how long in 24 hours is the sun above the horizon.

8. Find the fluxion of $\frac{x}{1+x^2}$, and the fluent of $\dot{x}(1+ax)^{-\frac{3}{2}}$.

9. 2, the focus of incidence, is distant 100 radii from the vertex of a concave spherical reflector; Find the place of q, the focus of reflected rays.

10. S, S', S'', are the sums of 3 arithmetical series; 1 is the first term in each, and the respective differences are 1, 2, 3; then

$S+S''=2S'$. Required proof.

11. $\ldots\ldots x^2+y^2=5,$
$\quad\quad 3xy\ \ =6.$

Find x and y.

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Evening Problems.—Mr. Woodhouse.

1. Required the amount on £30. 15s. at 3*d.* in the pound.

2. The first term in an arithmetical series is 1, and the difference is m; then the sum of n terms $= \frac{mn^2 - (m-2)n}{2}$. Required proof.

3. Let $x^2 - px - q = 0$, assume tan. $z = \frac{2\sqrt{q}}{p}$, rad. $= 1$; then the two roots of the quadratic are, $\frac{p}{2}$tan. z. cotan. $\frac{z}{2}$, and $-\frac{p}{2}$tan. z. tan. $\frac{z}{2}$. Required proof.

4. Prove, that in every quadrilateral figure, the sum of the squares of the two diagonals is double of the sum of the squares of the two right lines that join the middle points of the opposite sides.

5. If £1. at compound interest, rate r, doubles itself in n years; and, rate $2r$ in m years; what is the ratio between n and m? Is it less or greater than, or equal to, the ratio of 2 to 1?

6. If 2 cos. θ be assumed $= u + \frac{1}{u}$, rad. $= 1$, then,

$$2 \cos. 2\theta = u^2 + \frac{1}{u^2},$$

$$2 \cos. 3\theta = u^3 + \frac{1}{u^3}$$

and generally, $2 \cos. n\theta = u^n + \frac{1}{u^n}$. Required proof.

7. Find the fluxion of h. l. $x . e^{\cos. x}$, e being the base of the hyp. logarithms.

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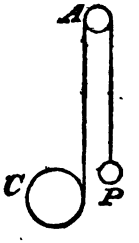
8. Find the fluents of $\frac{\dot{x}x}{(1+x)^{\frac{1}{2}}}$, and of $\frac{\dot{x}}{1+a.\cos.x}$, $a \angle 1$.

9. A pipe, cylindrical within, is placed vertically, filled with a fluid: What ought to be the law of the increase of the thickness of its sides, that it may be equally strong throughout; the increase of strength being supposed to vary with the increase of thickness?

10. Let two bodies, with velocities v, v', be projected at the same time and towards each other, from the two extremities of a vertical line; then they shall always meet in the middle of the line, if the difference of the squares of the velocities v, v', is equal to the square of a velocity acquired in falling down half the line.

11. The weight P is attached to a string which goes over a fixed pulley at A, and is wound round a thin cylinder C, the weight of which is W. Required P, such, that it shall neither ascend nor descend, whilst C descends by the unwinding of the string.



12. If from the four angles of a pyramid there be drawn lines to its centre of gravity, then a point there placed, shall be kept at rest by forces represented by the 4 lines. Required proof.

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13. Let the axis of motion pass through a thin straight rod of uniform thickness and density, dividing the rod into 2 arms that have the ratio of 2 to 1: It is required to find the velocity acquired by the extremity of the longer arm, when the rod shall have moved from an horizontal to a vertical position.

14. If the sine of incidence be to the sine of refraction as $1 : n$, then the focal length (f) of a double convex lens, may be nearly had from this expression,

$$\frac{1}{f} = \frac{1-n}{n}\left\{\frac{1}{r} + \frac{1}{r'} + \frac{1-n}{r^2}t\right\},$$

r, r' being the radii of the two surfaces, and t a very small quantity, the thickness of the lens. Required proof.

15. Prove, that the earth's angular velocity at mean distance is less than the mean angular velocity.

16. Prove, that the augmentation of the sun's apparent diameter from apogee is nearly proportional to the versed sine of angular distance from apogee; the excentricity of the earth's orbit being a small quantity.

17. Given the right ascension and declination of the moon's centre, and of a fixed star; Find their distance.

18. Given the latitude of a place, the sun's declination and altitude; Find the azimuth, and under a form adapted to logarithmic computation.

19. The line of the nodes being in octants, and the body P. (NEWT. Sect. 11.) in quadratures, compare the disturbing force that acts perpendicularly to the plane of the orbit, with the like force, when the line of the nodes is in quadratures, and the body in

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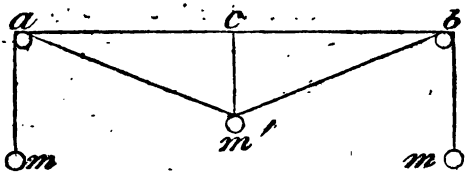
octants; the inclination of the orbit being supposed to be the same.



20. a, b, are two fixed pullies over which the string passes, to which m, m', m, three equal weights, are fastened: Suppose m' to be let fall from c, it is required to find the distance from c, at which its velocity is the greatest.

21. Let a weight P, fastened to a string going over a wheel, by its descent, cause two weights, 2, $2'$, to be wound up on two axles. Required the velocity of P, after that it has descended through a space s; the radii of the wheel and of the two axles being r, r', r''.

22. A and B play at a game, each with an equal chance of success, and with this condition; A gives to B a sum S if he loses the first time; $2S$, if he loses two games successively; $4S$, if three games successively, &c.; and $2^{n-1}S$, if he loses n games successively: What sum, on the received principles of chances, ought B to give to A, as an equivalent of the risque he encounters?

Tuesday Morning Papers.—Mr. Woodhouse.

FIRST AND SECOND CLASSES.

1. Given, the logarithm of $8.1213 = .9096256$;

It is required to state the logarithms of $\left\{\begin{matrix} 81.213 \\ 812.13 \\ .81213 \\ .081213 \end{matrix}\right\}$

and to prove the justness of the operation.

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2. Prove that

$$\tan.(45+A)=\tan.(45-A)+2\tan.2A.$$

3. Prove that the tangential part of the ablatitious force varies as sin. 2θ, θ being the angular distance of P, from quadratures (NEWT. Sect. 11.)

4. The force varying as the distance, let a body be projected with a certain velocity, at a given distance *(d)* from the center, at an angle, the sine of which is S; It is required to prove that the orbit described must have an apse.

5. State the usual distinction between apparent solar year, and mean solar year; and explain the cause of their difference.

6. Explain the method of finding the right ascension and declination of a star.

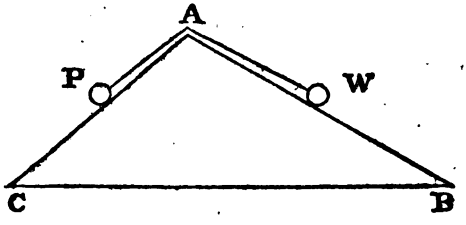
7. Given the right ascension and declination of a star; Find its longitude.

8. Prove that $\int\frac{\dot{z}}{(\cos.nz)^2}=\frac{1}{n}\tan.nz.$

9. If the h. log. of n be given, then the h. log. $(n+1)$ may be conveniently found from this formula:

$$\text{h.log.}(1+n)=\text{h.log.}\,n+2\left\{\frac{1}{2n+1}+\frac{1}{3}\cdot\frac{1}{(2n+1)^3}+\&c\right\}$$

Required proof.



10. Given the plane AC; It is required to find the elevation of the plane AB, such, that W shall be

drawn by P from the horizontal line BC to the point A in the least time possible.

11. Find the integral of $\frac{4z+3k}{z.(z+k)(z+2k)}$, k being the increment of z.

12. Let $y=\frac{x}{1+x^2}$; trace the curve, find the values of y when a maximum, and at the point of contrary flexure; and find the value of the angle in which the curve cuts the axis at the point of intersection.

13. Investigate a formula for clearing the moon's distance from a star, from the effects of parallax and refraction.

Morning Problems.—Mr. D'Oyly.

THIRD, FOURTH, FIFTH, AND SIXTH CLASSES.

1. What is the discount on £80. 10s. for 3 months, at $3\frac{1}{2}$ per cent.?

2. A clock has two hands, turning on the same centre; the swifter makes a revolution in 5 hours, the slower in 9. If they start from the same point, when will they next meet?

3. Prove NAPIER'S rules for solving a right-angled spherical triangle, in the two cases, in which one of the sides is made the middle part.

4. Prove, that the tan. + cotang. of an arc is equal to twice the cosec. of double the arc.

5. The roots of $x^3-7x^2+14x-8=0$ are in geometrical progression. Find them.

6. Find a number greater than the greatest positive root, and also one less than the least negative root of the equat. $x^3-4x^2-x+20=0$.

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7. A lighter fluid, whose specific gravity is 3, rests upon a heavier, whose specific gravity is 7. A paraboloid, with its vertex downwards, rests between them, and sinks with $\frac{1}{4}$ of its axis in the heavier.—Find its specific gravity.

8. Two bodies, *P* and 2, each 1 lb. in weight, balance on a single pulley. An ounce weight is added to *P*. How long is it in descending through 12 feet? and what velocity does it acquire?

9. A set of 5 balls, imperfectly elastic, are in a geometrical progression, whose common ratio is 2. The force of elasticity is to the force of compression :: 3 : 2. Compare the velocity of the first with that communicated to the last.

10. The tube of a barometer is 33 inches long. A quarter of an inch of air was left in at the time of inversion. When the mercury in this barometer stands at 28 inches, what is the true standard altitude?

11. Find the latitude of the place, at which the sun sets at 10 o'clock on the longest day.

Tuesday Afternoon.—Mr. D'Oyly.

1. Find two numbers, such, that their difference may be 4, and that twice their product may equal the cube of the less.

2. A curve has its subnormal : $\overline{\text{abscissa}}^3$ :: $2a^2$: 1. Find the equation to the curve; draw a tangent to it, and find its area.

3. Find the focal length of a double convex lens; and shew that in a glass lens, with surfaces equally curved, it is equal to the radius of either surface.

4. When the moon's elongation is 60°, delineate her phase, and prove the area of the enlightened part to be $\frac{1}{3}$ the area of the dark part.

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5. A comet is in the perihelion of a given ellipse. Compare its velocity with the velocity it would have in a parabola, at the same perihelion distance.

6. An object is placed before a double concave lens, at the distance of 5 feet, and has the linear magnitude of its image 7 times less than its own.—Find the focal length of the lens.

7. Sum the series $\frac{1}{3}-\frac{1}{3.2}+\frac{1}{3.2^2}-$ &c. ad infin. also sum n terms of $1^2+2^2+3^2+4^2+$ &c.

8. Find the fluents of $x^3\dot{x}\sqrt{1-x^2}$; of $\frac{x\dot{x}}{\sqrt{1-x^4}}$, and of $\frac{z\dot{z}}{1+2az+z^2}$, where a is less than unity.

9. Find the relation of x to y in $\sqrt{1+y^2}\times\overline{xy+y\dot{x}}=y\dot{y}$.

10. If the 4th power of the periodic times in different circles varies as the cube of the velocities, find how the force, periodic time, and velocity, vary in terms of radius.

11. When a body describes the reciprocal spiral, prove that it makes equal approaches to the center in equal times; and find the area contained between any two distances.

12. In what part of the moon's orbit does the ablatitious force bear to the addititious the proportion of 4 to 3?

Fifth and Sixth Classes.—Mr. D'Oyly.

1. Find the greatest common measure of 720,336, and 1736.

2. Prove the rule for finding the square root of a binomial surd, and extract the square root of the surd $5-2\sqrt{6}$.

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3. Shew that in a plane triangle, the base : sum of sides :: difference of sides : difference of segments of base.

4. Find the value of x in $3x+2\sqrt{x}-1=0$.

5. An inclined plane has its length 6 times its height: How long will a body be in falling through 20 feet? and what velocity will it acquire?

6. A body weighing 6 lbs. in air, weighs 2 lbs. in water; another weighing 7 lbs. in air, weighs 4 lbs. in water. Compare their specific gravities.

7. Prove that the periodic times in all ellipses round the same center are equal.

8. Sum $\frac{2}{10}-\frac{2}{100}+\frac{2}{1000}-$ &c. ad infin.

and $5+7+9+11+$ &c. to 50 terms.

9. Find the fluxion of $\frac{x}{\sqrt{1+x^2}}$, and the fluent of $\frac{a\dot{x}}{\sqrt{2ax-x^2}}$.

10. A short-sighted person, who cannot see distinctly beyond 3 feet, wishes to see an object at 14 feet distance. What sort of glass must he use, and what must be its focal length?

Evening Problems.—Mr. D'Oyly.

1. Find the exact value of the decimal .012323 &c. of a pound.

2. Let $x^2y^2+6xy=16$, and $\sqrt{\frac{1}{4x}}=\frac{1}{2}\sqrt{\frac{1}{y-1}}$. Find corresponding values of x and y.

3. The roots of $8x^3-6x^2-3x+1=0$ are in harmonical proportion. Find them.

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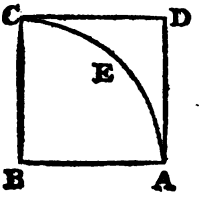
4. Prove that the perpendicular ascent of an heavenly body is always quickest on the prime vertical.

5. A cylindrical vessel of given diameter is filled with fluid to a certain height; its height is divided at some point. What must be the height of the fluid, and where the point of division, so that the pressures on the base, on the upper surface, and on the lower, may all be equal?

6. Find the attraction of a particle placed in the vertex of a paraboloid, the attraction on each particle being reciprocally as $\overline{\text{dist.}}^2$.

7. The equation to a curve is $y\sqrt{2ax-x^2}=ax$. Find its area; and prove, that when x is equal to a, the whole area equals the excess of $ABCE$ above $ADCE$; AEC being a quadrant, and $ABCD$ a square on AB.



8. A body weighing 2lbs. is projected with a velocity 20, at an angle of 60°; another, weighing 3lbs. is at the same time projected at an angle of 30°, with a velocity 25. Trace the motion of their common center of gravity; find the height to which it mounts, and the distance at which it strikes the horizontal plane.

9. AEC is a quadrant; the curve $A2m$ is traced by taking its abscissa always = arc AE, and its ordi-

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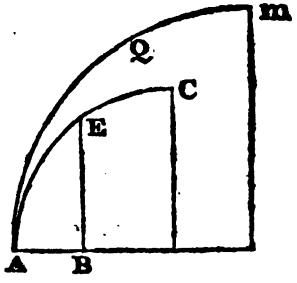
nate = twice sine EB. Draw a tangent to the curve at any point, and find its area for the whole quadrant.



10. The chances that each of two individuals will die in a certain time are respectively $\frac{5}{6}$ and $\frac{4}{5}$. Find the two probabilities, that they will not both be alive, and that they will not be both dead, at the end of that time.

11. Explain the method of transforming an equation, in which x flows uniformly into one, in which y flows uniformly; and by this method solve the equation $\dot{x}\dot{y} - x\ddot{y} = x\dot{y}^2$.

12. A weight, suspended by a string of given length, is whirled round in a conical motion; that is, it describes, parallel to the horizon, the base of a cone, of which the vertex is the point of suspension. The tension of the string being 3 times the natural weight of the body, in what time does it revolve?

13. A body is projected from an apse with a velocity, which bears to the velocity, in a circle at the same distance, the proportion of 1 to $\sqrt{3}$, force varying as $\frac{1}{D7}$. Find the angle which it describes round the centre of force, before it falls into it; and

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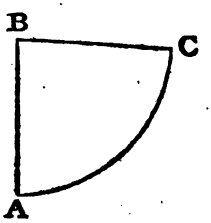
compare the time with the periodic time in a circle at the same distance.

14. A straight line is placed at the distance of 9 inches before a double convex lens, whose focal length is 6 inches. Prove that the image will be a conic section, and determine its major and minor axis.

15. A cylindrical bent tube is partly filled with a fluid, whose surfaces rise and fall alternately. Shew that this fluid librates twice, in the time that a pendulum, whose length equals twice the line of fluid, oscillates once.

16. A chain of uniform thickness, hanging freely, and pliant in every part, forms itself into a cycloid. Required the law of its density and weight.

17. The quadrant AC is a thin rod, which revolves on the axis AB perpendicular to the horizon. A ring moves freely on the rod. With what velocity must the point C revolve, in order that the ring may always remain at the middle of the arc?



18. A pendulum, vibrating in a certain time at the pole of a sphere, vibrates once less in 100 times, when carried to a place 30° from its equator. In what time does the sphere revolve round its axis?

19. Given a star's aberration in latitude and long.; Find the aberration, in declination and right ascension; and explain where a star must be situated, so

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as to have the same aberration in latitude and declination.

20. Two globes, of equal weights and diameters, are placed in contact, on a straight rod which passes through both their centers; the point of suspension is distant one diameter from the vertex of the upper one; Find their common center of oscillation.

21. Find fluents of $v^3x^2\dot{x}$, where v is h. log. of x; of $\frac{\dot{x}}{x\sqrt{1+\sqrt{x}}}$, and of $v\dot{x}\dot{x}$, where $v=$ h. l. $\frac{a+x}{a-x}$.

22. Sum the following series:

$$\frac{1}{2}-\frac{1}{2.2^3}+\frac{1}{2.2^6}-\text{\&c. ad infin.}$$

$$\frac{2}{1.3.4}+\frac{3}{2.4.5}+\frac{4}{3.5.6}\ \text{\&c. to } n \text{ terms;}$$

and, $1^2+4^2+7^2+10^2\ldots$ to n terms.

23. The powers of the sun and moon to raise a tide being respectively given, and the moon's elongation from the sun; Find the place of the compound high tide.

24. A man descends from an height in the atmosphere, suspended from the center of a plane circle, whose diameter is d. The weight (in air) of himself and the machine is w lbs.; a cubic foot of air weighs every where $1\frac{1}{5}$ oz. and gravity is constant; Find the greatest velocity he can acquire, and shew that he can attain this velocity in no finite time.

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1808.

First Morning.—Mr. D'Oyly.

MONDAY, JANUARY 18.—FIRST AND SECOND CLASSES.

1. Prove that the difference of the tangent and co-tangent of an angle is equal to twice the co-tangent of double the angle.

2. Prove Napier's rules for the solution of a right-angled spherical triangle, in the two cases, in which the complement of the hypothenuse is made the middle part.

3. When a pendulous body oscillates in a cycloid, beginning at the highest point, compare the tension of the string at the lowest point, with the tension caused by the natural weight of the body.

4. Find the actual velocity and periodic time of a body revolving at the distance of two of the earth's radii above its surface.

5. A straight rod is suspended at a point, distant $\frac{1}{4}$th of its length from its extremity. Compare the time of its oscillation in this case, with the time of its oscillation when it is suspended at its extremity.

6. Given $a\dot{z}=\overline{by+a^2}|^{\frac{1}{2}}\dot{y}$, where z is the arc of a curve, and y the ordinate; find the relation between the ordinate and abscissa.

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7. Find the centre of gravity of a given sector of a circle.

8. If two bodies are connected by a lever void of gravity, and a force acts, perpendicular to the line which joins them, at a point which is not their centre of gravity; find the centre of initial rotation.

9. A cylinder, whose radius = 2 feet, and whose weight = 100 lbs. is moveable on its axis. A weight of 1 lb. is suspended at the distance of half its radius from the axis. How long must this weight act, to generate a motion of 10 revolutions in 1″?

10. Affect $\overset{v}{-3}\overset{v}{-1}\overset{v}{2}$ with v, and reduce it to succeeding values of v.

11. Find fluents of $\overline{\sin.z}\dot{z}$, of $\frac{x^m\dot{x}}{\overline{\log.x}}$, and of $\frac{\dot{x}}{1-x^4}$.

12. Sum $\frac{5}{1.2}\times\frac{1}{3}+\frac{7}{2.3}\times\frac{1}{9}+\frac{9}{3.4}\times\frac{1}{27}+$ &c. to n terms;

and $\frac{1}{1^3}-\frac{1}{3^3}+\frac{1}{5^3}-$ &c. ad inf.

13. Compare the chance of throwing, with a single die, an ace at least 3 times in 5 trials, with the chance of throwing it 3 times exactly in the same number of trials.

First Afternoon.—Mr. Woodhouse.

1. Compare the interest on £350. 15s. at 4½ per cent. with the interest on £450. 15s. at 3½ per cent.

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2. Prove, 1st. That three perpendiculars from the angles of a triangle upon the opposite sides intersect each other in the same point.

2dly, That three lines drawn from the three angles of a triangle, and bisecting the opposite sides, intersect each other in a point which is the centre of gravity of the triangle.

3dly, That three lines drawn from the three angles of a triangle, and bisecting those angles, intersect each other in a point, which is the centre of an inscribed circle.

4thly, That three lines bisecting the three sides of a triangle, and drawn perpendicularly to the sides, intersect each other in a point which is the centre of a circumscribed circle.

3. In what point of the cycloid will the force stretching the string, and the force accelerating the body, be equal?

4. If the perihelion distance of a comet moving in a parabolic orbit be 64, the earth's mean distance being 100, compare its velocity at the extremity of the latus rectum with the earth's mean velocity.

5. Compute the value of the tangent of 15°.

6. By what method can the planets' distances from the sun be computed?

7. Find the fluxion of $\left(1-x^{\frac{1}{2}}+x^{\frac{1}{3}}\right)^{\frac{4}{5}}$

and the fluent of $\frac{\dot{x}+\frac{1}{3}x\dot{x}}{\sqrt{1+x^2}}$.

8. Place an object 100 feet before a concave spherical reflector, of which the radius is 10 feet; then find the distance between the principal focus and the focus of reflected rays.

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9. Define the longitude of a place, and explain by what methods it can be determined.

10. Find the present value of a perpetuity of £5. per annum, to be received at the end of seven years, the rate of interest being 5 per cent.

11. Investigate an expression, from which, a pendulum losing or gaining n seconds in 24 hours may be shortened or lengthened.

12. Explain what is meant by the Aberration of Light?

If from such aberration we perceive the apparent places only of stars, by what means can their real places be assigned?

13. What is the difference between the latera recta of a parabola and ellipse having the same least distance $=1$, the axis major of the ellipse being 300?

14. Given the logarithms of $20=1.30103000$
of $30=1.47712125$
Deduce thence the logarithm of 360.

15. Explain the construction of the horizontal sun-dial; and compute, for the latitude of Cambridge, the hour-angle between 2 and 3.

16. If bodies are imperfectly elastic, shew that the sum of the product of each body into the square of its velocity, before impact, is greater than the sum of the product of each body into the square of its velocity after impact.

17. If the sun's apparent diameter at the extremity of the latus rectum were to his apparent diameter at perigee as 100 ; 101, what would be the excentricity of the solar ellipse?

18. Define the centrifugal force; find an expression for the law of its variation; and compare it

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with the centripetal force which acts upon a body moving in the logarithmic spiral.

19. Find the nature of the curve, in which the square of the ordinate is a mean proportional, between the area and a given quantity b.

20. Let a body be projected at an angle of 45° with a velocity acquired down the axis of a cycloid. Compare the time of flight with the time of an oscillation.

21. Sum the arithmetical series

$$1+5+9+13 \text{ \&c. to } n \text{ terms.}$$

22. State Napier's rules; and prove their truth, in the case in which the complement of the hypothenuse is the middle part.

23. Find the attraction of a corpuscle situated in the axis of a cylinder towards the cylinder, the particles of which attract according to the law of the inverse square of the distance.

Evening Problems.—Mr. Woodhouse.

1. Define discount; investigate a formula for computing it; and apply such formula to the computation of the discount on £100. due at the end of 3 months.

2. Find the sum of n terms of a series of polygonal numbers, which numbers are formed, by assuming any arithmetical series that has its first term 1, and difference a whole number, and by making, generally, the m^{th} polygonal number equal to the sum of m terms of the arithmetical series.

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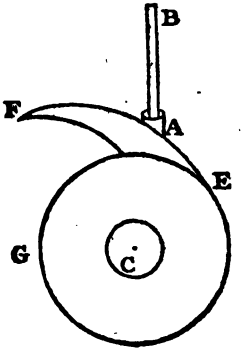
3. By Nautical Almanack, the moon's right ascension was at Midnight, March 10, = 4°. 56′,
Noon, March 11, = 10°. 34′,
Midnight, March 11, = 16°. 18′.
It is required to find the right ascension March 11, 3^h.

4. The axis of the solar ellipse having a yearly progressive motion of 1′. 2″, and the longitude of the perigee being in 1750, 278°. 37′. 15″; it is required to find the epoch, at which, the axis was perpendicular to the line of the equinoxes.

5. From the two ends of a vertical line, two bodies are, at the same instant, projected towards each other, with velocities v, v'. Required their distance, when half the time, in which they would meet, is elapsed.

6. If a power P descending vertically by means of a string passing over a fixed pulley, elevates a weight W; what will be the pressure sustained by the axis of the pulley?

7. Let the wheel EG revolve round an axis at C. Required the nature of its tooth or *wing* EF, that shall elevate vertically the piston AB through equal spaces, whilst the wheel revolves through equal angles.



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8. Given the velocity of sound; find the horizontal range when a ball at a given angle is so projected towards a person, that the ball, and sound of the discharge, reach him at the same instant.

9. The late comet passed its perihelion Sept. 21, 1807; and the perihelion distance was .64, the earth's mean distance being 1. Required the computation of the velocity with which it is now moving, January 18, 1808; the orbit being supposed to be parabolic.

10. With three dice in one throw, compare the chance of throwing the number 10 with the chance of throwing the number 5.

11. Find the fluent of the fluxional equation $\frac{\ddot{y}}{\dot{t}^2} + a^2 y = 0$.

and of $\dot{\theta} \sin. m\theta . \sin. n\theta .$ (radius $= 1$.)

10. Let a square be immersed in a fluid, with its plane perpendicular to the surface. Compare the pressures on the two halves of the square, made by drawing its diagonal.

13. If the fluent of the fluxional equation $p\dot{x} + q\dot{y} = 0$ can be assigned, p and q being functions of x and y, then $\frac{p}{\dot{y}} = \frac{q}{\dot{x}}$ Required proof.

14. Compare the velocity of the moon in octants, with her velocity in syzygies, according to the principles of the 11th section.

15. Find, by the method of increments, the sum of

$$\frac{1}{1.2.3.4} + \frac{3}{2.3.4.5} + \frac{6}{3.4.5.6} + \frac{10}{4.5.6.7} + \&c.$$

the numerators being the second order of figurative numbers.

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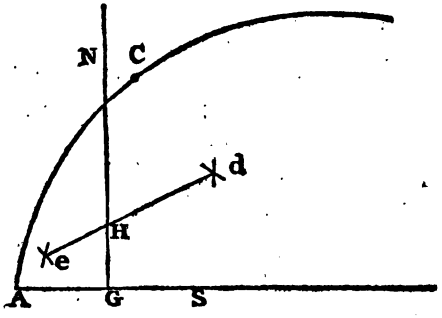
16. If C be the place of a comet in its parabolic orbit, A the perihelion, S the sun, G the middle point between A and S, GN a perpendicular to AS; then, if from the centres C and S, and with any radius, circular arcs be described intersecting each other in e and d, and ed be joined, a point H in GN is determined, such, that GH shall be proportional to the time of the comet's moving through the arc AC. Required proof.



17. Let d be the distance of the focus of incidence from the centre of a concave reflector, and let a ray be incident on the reflector 60° from its vertex. It is required to find the distance, in the axis of the reflector, between the geometrical focus and the intersection of the reflected ray with the axis.

18. If a pendulum, vibrating through an arc of 2 degrees on each side of the vertical, keep true time, find nearly the error of time introduced by making it vibrate through 2°. 10′.

19. Express the cos. $4x$ in terms involving powers of cos. x.

20. Investigate the nature of the curve, of which the normals bear the same relation to the parts of the

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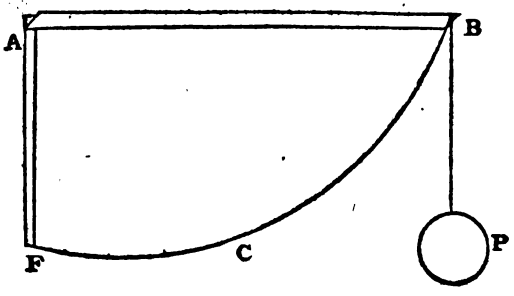
abscissa intercepted between the origin of the abscissas and normals, that the ordinates of a common parabola bear to their corresponding abscissas.

21. Suppose a piston, closely fitting a vertical tube, to descend by its weight, the tube being filled with an elastic fluid, and closed at the end opposite to the piston; Find the velocity of the piston after a descent through a space S, not considering the effect of friction.

22. If a body be made to descend obliquely along the concave surface of a vessel, the axis of which is perpendicular to the horizon; then the fluxion of the area, projected upon a plane perpendicular to the axis, is a constant quantity. Required demonstration.

23. $BCFA$ is a piece of timber inserted into a wall at AF. If a weight P be appended to the end B, what ought to be the form of the under-bounding surface BCF, that the strength of the beam may be every where the same; the weight of the beam being supposed to have no effect in producing fracture?



24. $x^3 - 2x^2 + 1 = 0$. Deduce the equation of which the roots are the cubes of the roots of the original equation.

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Tuesday Morning Problems.—Mr. Woodhouse.

FIRST AND SECOND CLASSES.

1. The rate of interest being 5 per cent. in what number of years, at compound interest, will £1. amount to £100? (Log. 1050 = 3.021 1893.)

2. Prove, that $\tan. 50 = \tan. 40 + 2. \tan. 10.$

3. Find the algebraic relation between v and z that satisfies this equation

$$\frac{\dot{v}}{\sqrt{(Av^2+Bv+C)}}+\frac{\dot{z}}{\sqrt{(Az^2+Bz+C)}}=0.$$

4. Prove, that

$$\text{h. log. } x = x - x^{-1} - \frac{x^2 - x^{-2}}{2} + \frac{x^3 - x^{-3}}{3} - \&c.$$

5. If $e^y = \frac{e^x+1}{e^x-1}$, find the length of the curve; y and x being its ordinate and abscissa, and e the number whose hyp. log. is 1.

6. Required the time of the passage of a comet from its perihelion, through 45° of anomaly; the perihelion distance being 1.

7. In the 10th section, if the diameter of the wheel be equal to the radius of the globe, the cycloid within the globe, or the *hypocycloid*, becomes a right line passing through the centre. Required proof.

8. Explain the principle and use of the common Vernier or Nonius.

9. Apply the method of increments to sum n terms of the series

$$\frac{1}{1.2.3}+\frac{2^2}{2.3.4}+\frac{3^2}{3.4.5}+\frac{4^2}{4.5.6}+\&c.$$

$$1 + 2.5 + 3.5^2 + 4.5^2 + \&c.$$

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10. If the sun's declination on the meridian of Greenwich

were, October 5, 4°. 28′. 36″,
and, October 6, 4°. 51′. 46″,

what would be the declination at 2 o'clock, October 5, at any other place 45° west of Greenwich?

11. Prove that the times of moving from the perihelion to the extremity of the latus rectum, in different parabolas, vary in the sesquiplicate ratio of the perihelion distances.

12. If, with a force varying $\frac{1}{(\text{dist.})^2}$, a velocity which is to velocity in circle as $\sqrt{3} : \sqrt{2}$, at an angle θ, and at a distance d, a body be projected; find the excentricity of the orbit described.

13. Let the luminous point be the extremity of the diameter of the circular reflector; find the length of the caustic, its cusp, and the density of rays in its cusp.

Fifth and Sixth Classes.—Mr. Woodhouse.

1. If $\frac{1}{4}$ of a yard cost 9s. 4d. what number of yards will £100. purchase?

2. Solve the equation $1 - \frac{2-x}{x} = \frac{x+2}{2} - 1$,

and of the two expressions x^3+y^3
x^2y+xy^2.

Shew which is the greater.

3. With what velocity must a body be projected from a tower, in a direction parallel to the horizon, so that it shall strike the ground at a distance from the foot of the tower, equal half the height of the tower?

4. What is the present value of the compound interest of £100. to be received five years hence?

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5. Compare the angular velocity of the earth at its mean distance, with its angular velocity in perihelion.

6. Find the fluxion of $\frac{z}{1+z}$, and the fluent of $x^{\frac{3}{5}}\dot{x}\left(1+ax^{\frac{3}{5}}\right)^{-\frac{1}{2}}$

7. Find the focal length of a spherical drop of water, the radius of which is $\frac{1}{10}$th of an inch.

8. If the periodic time of Mercury be to the periodic time of the earth as 4 : 11, required the time from conjunction to conjunction.

9. In what time will a weight move an equal weight along an inclined plane, the length of the inclined plane being 12 feet, and the height 10 feet?

10. Required the specific gravity of a body that floats in 2 fluids, of which the specific gravities are a and b, when the n^{th} part of the body is in the upper lighter fluid.

11. Given, in a rectilinear triangle, two sides and the included angle; find the third side.

Tuesday Afternoon.—Mr. D'Oyly.

THIRD, FOURTH, FIFTH, AND SIXTH CLASSES.

1. Find the exact value of the circulating decimal .212121 &c.

2. Find a number consisting of 2 places, such, that when it is divided by the difference of its digits, the quotient is 21; and that, when it is divided by the sum of its digits, and the quotient increased by 17, the digits are inverted.

3. The roots of the equation $x^3-14x^2+56x-64=0$ are in geometrical progression. Find them.

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4. Given the tangents of two arcs, find the tangents of their sum and difference.

5. Shew, that in a right-angled spherical triangle, if the legs are of the same affection, the hypothenuse is less than a quadrant; and, if the legs are of different affection, the hypothenuse is greater than a quadrant.

6. A body, projected at an angle of 15° to the horizon, ranges 100 feet on an horizontal plane.—How high would it rise, if it were projected straight upwards, with the velocity of projection?

7. The barrel of a condenser, and each barrel of an exhausting air-pump, is $\frac{1}{10}$th of the receiver, and the receiver is equal in both. Let each be worked 3 turns. How much is the density increased in the former, and how much is it diminished in the latter?

8. Prove, that parallax depresses an heavenly body in a vertical circle, and that the parallax at any altitude varies as the sine of the apparent zenith distance.

9. Let a point be taken in the axis of a given ellipse, half-way between its centre and focus. Find the shortest line from this point to the circumference.

10. Prove, that if a body could fall from the surface of the earth to its centre, it would acquire a velocity equal to that with which it would revolve in a circle at the surface; and find the actual velocity.

11. Find the fluents of $\frac{\dot{x}}{1+x^2}$, $x^3\dot{x}\sqrt{1+x^2}$, $\frac{\overline{a^2+x^2}|^{\frac{1}{2}}\dot{x}}{a}$, and $a^x x^2 \dot{x}$.

12. Prove that the mean addititious force in quadrature is to the moon's whole gravitation to the earth, as the square of the moon's periodic time to the square of the earth's periodic time.

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13. Compare the side of an equilateral triangle inscribed in a circle with the radius of the circle.

14. Given the sine of 1′; explain how the sines of 2′ 3′, &c. may be found.

15. Solve the quadratic equation $5x=3-2x^2$.

16. Find the 4 roots of the recurring equation

$$x^4-\frac{5}{2}x^3+2x^2-\frac{5}{2}x+1=0.$$

17. Deduce the equation to a parabola, and prove it to be a conic section.

18. A body is projected up an inclined plane, whose height is $\frac{1}{6}$th of its length, with a velocity of 30 feet per second. Find its place, and its velocity after 12 seconds.

19. A cylinder, whose height is 3 times its diameter, is filled with a fluid. Compare the pressure on the sides with the pressure on the base.

20. An object placed 4 inches before a double convex glass lens, has its image formed 9 inches from the lens on the same side. Required the focal length of the lens.

21. Find the attraction of a particle placed in the vertex of a cone, the attraction of each particle being inversely as $\overline{\text{dist.}}^2$

22. If the force varies inversely as $\overline{\text{dist.}}^2$, how far must a body fall externally and internally, to acquire the velocity in a circle?

23. Sum series $\frac{1}{2}-\frac{1}{2.3}+\frac{1}{2.3^2}-\frac{1}{2.3^3}+$ &c. ad infin.

$$\frac{1}{2.4.6}+\frac{1}{4.6.8}+\frac{1}{6.8.10}$$ to n terms.

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24. Trace Cotes's first spiral; compare the velocity of a body in it with the velocity in a circle, and the time of its falling to the centre with the periodic time in a circle.

Fifth and Sixth Classes.—Mr. D'Oyly.

1. What fraction of half-a-crown is $\frac{1}{7}$ of $\frac{2}{11}$ of a guinea? and what is its exact value?

2. Extract the square root of $2-4\sqrt{-1}$.

3. What number is that, the treble of which, added to the double of its square root, is equal to 1.

4. Given the sines and consines of two arcs; find the sine and cosine of their sum and difference.

5. Prove Cardan's rule for solving a cubic equation; and shew that it can only be applied where two roots are impossible.

6. A body is projected up an inclined plane, whose length is 10 times its height, with a velocity of 30 feet in 1″. In what time will its velocity be destroyed?

7. A plane surface moves in a fluid, in a direction perpendicular to its plane, with a certain velocity. Another plane, 4 times as large, moves in a fluid of 3 times the density, and with twice the velocity. Compare the resistances.

8. Given the latitude of the place, and the sun's declination; find the time of its rising.

9. Find the fluxion of $\frac{\overline{a^2-y^2}^{\frac{1}{2}}}{y^{\frac{1}{2}}}$, and fluents of $\frac{\dot{y}}{1+y}$ and $\frac{2b\dot{x}}{x\sqrt{x^2-b^2}}$.

10. Deduce the general expression for the centripetal force; and apply it to find the variation of the force in the logarithmic spiral.

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11. Sum $\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}+$&c. to inf.

$1+2+3+4+$&c. to n terms.

12. Shew, that when the force $\propto$ dist. all times of descent to the centre are equal; and that the velocity at any point is as the sine of the arch, whose versed sine is space described, the first distance being radius.

Evening Problems.—Mr. D'Oyly.

FIRST, SECOND, THIRD, AND FOURTH CLASSES.

1. Prove the rule for extracting the square root of a binomial surd; and apply it to extract the square root of $1-4\sqrt{-3}$.

2. The sun's declination being 16° N. required the latitude of the place where it rises in the N. E. point, and also the time of its rising.

3. P and Q, each weighing 2 lbs. balance one another over a pulley; 1 oz. is added to P. How far will it have descended in 5″, and what velocity will it have acquired?

4. Compare the density of light in the sun's image, formed by a sphere of water, and by a plano-convex glass lens of twice the aperture and twice the radius, the transmitting powers being supposed the same.

5. Find the fluents of $\frac{dx^2\dot{x}}{\sqrt{1-x^2}}$ of $\frac{\dot{v}}{\sqrt{a^2+v^2}}$, and of $z^3\dot{y}$, where z is the circular arc, whose sine is y to radius a.

6. A vessel, whose shape is the frustum of a cone of given dimensions, is filled with a fluid, the smaller end being downwards, and is turned round on its axis with such a velocity, that all the fluid flies out. Required the velocity with which it turns round.

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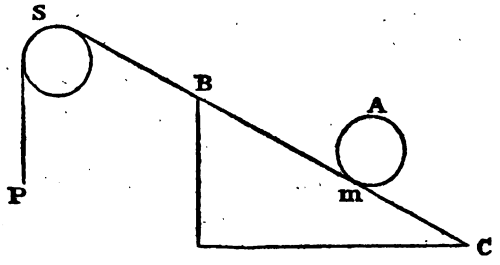
7. Approximate to a value of x in $x^3 - x - 50 = 0$; and shew on what the accuracy of an approximation depends.

8. While a body is revolving in a circle $\left(F \propto \frac{1}{D^3}\right)$ the absolute central force is suddenly doubled. Find in what time it will fall to the centre; and, when it has fallen half way, compare its velocity with that in a circle at the same distance.

9. The tangent of the angle, which the hour-lines for 3 and 4 o'clock make with one another on an horizontal dial, is to the tangent of the angle, which the hour-lines for 2 and 3 o'clock make with one another, in the given proportion of m to n. Find the latitude for which the dial is constructed.

10. A sphere A, whose weight is w, is made to roll down an inclined plane BC, by means of a string wrapt round it, which is sustained by P, $P = \frac{w}{10.}$ What must be the proportion of the height of the plane to its length, that P may neither ascend nor descend?



11. When parallel rays fall on a spherical refracting surface of given radius, the ratio of the sines of incidence and refraction being known, find the longitudinal and latitudinal aberrations.

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12. A curve is described, whose ordinate is the arc of a given circle, and whose abscissa is the corresponding sine. Find its area.

13. A thin rod of given length, weighing 1 lb. is suspended from its upper point, and has a weight $\frac{1}{4}$ lb. attached to it at $\frac{1}{3}$rd of its length from the point of suspension. Compare the time of its oscillation with this weight attached, with the time of its oscillation without it.

14. If the compressive force of the atmosphere varied as the logarithm of the density, and the density varied inversely as the distance, what would be the law of gravity?

15. If z be an arc of a circle to radius 1, shew that $\frac{1}{2}z = \sin. z - \frac{1}{2}\sin. 2z + \frac{1}{3}\sin. 3z -$ &c.

16. Integrate the fluxional equation $3x^2\dot{y} - 3ax\dot{y} = ay\dot{x}$.

17. Prove that

$$a^x = 1 + \frac{A}{m} + \frac{2AX + A^2}{2m^2} + \frac{3AX^2 + 6A^2 + A^3}{6m^3} + \text{&c. } A$$

and X being logarithms of a and x to modulus m.

18. A and B are playing a set at tennis. A wants two games, and B wants three; but A's skill is to B's in the proportion of 2 to 3. Find their respective chances of winning the set.

19. Find the integral of $\frac{1}{z}$.

20. Trace the curve, whose equation is $\sqrt{x^2 + y^2}^3 = x^2 - y^2$; find the value of x, when the curve is parallel to the axis; and determine the angle at which it cuts the axis, when the value of x is greatest.

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21. A body moves in a logarithmic spiral in a resisting medium $\left(\text{density} \propto \frac{1}{\text{dist.}}\right)$ with a force varying as $\frac{1}{\text{dist.}^{\frac{1}{2}}}$. It makes one revolution in t'', and approaches to the centre by 100th part of its first distance. Find the whole time of falling to the centre.

32. Sum series

$1.4.5+2.5.6+3.6.7+$ &c. to n terms;

$\frac{10}{1.2.4}+\frac{14}{2.3.5}+\frac{18}{3.4.6}+$ &c. to n terms;

$\frac{1}{1.4}-\frac{1}{3.6}+\frac{1}{5.8}-$ &c. to inf.

23. The arch of a bridge being a cycloid, what must be the nature of the curve bounding the wall above, that all the parts of the arch may be in equilibrio?

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1809.

First Morning.—Mr. D'Oyly.

MONDAY, JANUARY 16.—FIRST AND SECOND CLASSES.

1. Prove that if A and B are two angles, $\frac{\cos. A \times \cos. B}{r} = \frac{1}{2}\cos.(A+B) + \frac{1}{2}\cos.(A-B)$.

2. Find the roots of the equation $x^3 - 6x^2 - 4x + 24 = 0$, which are in arithmetical progression.

3. Given the horizontal parallax of an heavenly body, its right ascension, and declination; Find its parallax in right ascension at a given time in a given latitude.

4. Compare the momentum of a circle revolving on one of its diameters with that of its circumscribing square.

5. Find the point of contrary flexure in a curve, whose equation is $x = \overline{\text{hyp. log. } y}|^3$.

6. A vessel in the shape of an hemisphere is filled with a fluid. Compare the time of its emptying itself through an orifice in its vertex, with the time of emptying through an equal orifice in its base.

7. Shew how the densities of planets may be compared.

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8. Find the equation of the curve in which the sub-tangent $= b + y$; b being a constant quantity, and y the ordinate.

9. Find the integral of $\frac{1}{r^2}$.

10. Find the centre of gyration of a spherical superficies revolving on one of its diameters.

11. Prove the sum of the series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} +$ &c. continued ad infin. to be infinite.

12. Find fluents of $\frac{\dot{x}}{x^2.(x+a)}$, of $\frac{x^2\dot{x}}{(x-2)(x-3)}$, and of $\frac{x\dot{x}\log.x}{\sqrt{a^2+x^2}}$.

13. There are four individuals, for each of whom the chance of dying in a certain time, is $\frac{1}{10}$. Compare the chance, that two of them at least will be dead in that time, with the chance that exactly two will be dead.

Monday Afternoon.—Mr. Brown.

THIRD AND FOURTH CLASSES.

1. What is the interest of £260. for 18 months, at $4\frac{1}{2}$ per cent?

2. What is the discount of the same sum, for the same time, at the same rate?

3. The arms of a bent lever are in the proportion of 2 : 1. What must be the angle at which they are inclined, that, when in equilibrio, the shorter arm may be parallel to the horizon?

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4. Given R and r the radii of the surfaces of a convex lens, and $m:n$ the ratio of the sines of incidence and refraction; Find the focal length, and shew it to be the same, which-ever way the rays come.

5. A pendulum gains n seconds a day. How far must it be elevated above the earth's surface to keep true time?

6. Solve the following equation: $x^3-x^2-8x+12=0$, which has 2 equal roots.

7. Find the respective bearings of London and Constantinople from each other.

8. Compare the velocity of a body at the vertex of a parabola, with the velocity in a circle at the distance of half the latus rectum.

9. Find how far a body must fall externally, to acquire the velocity in an ellipse, when the force tends to the focus.

10. Find the sum of the following series: $\frac{1}{1.3}+\frac{1}{2.4}+\frac{1}{3.5}+$&c. ad inf. and n terms of $1^2+2^2+3^2$ &c. by the method of increments.

11. Find the fluxion of ay^x; and the fluents of $\frac{\dot{x}}{\sqrt{a^2+x^2}}$, and of $\frac{x\dot{x}}{\sqrt{2ax-x^2}}$.

12. Find the area of the common parabola intercepted between the ordinates b and c.

13. Shew, generally, in the conic sections, that $\frac{QT^2}{QR}=L$.

14. A body is projected with a velocity less than that acquired from an infinite distance, the force

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varying inversely as the cube of the distance. Investigate the orbit, and find in what time the body will fall into the centre.

Fifth and Sixth Classes.—Mr. D'Oyly.

1. If $2\frac{1}{4}$ yards of cloth cost 3s. $8\frac{1}{2}$d. what will be the price of $7\frac{2}{3}$ yards?

2. Extract the square root of $1+4\sqrt{-3}$; and prove the rule by which it is done.

3. Find the value of x in $3x-\frac{x-4}{4}-4=\frac{5x+14}{3}$.

4. Find two numbers such, that their product may be 24, and the sum of their squares 52.

5. Given the sine of an arc $=s$ to radius 1; express its cosine, versed sine, secant, tangent, and co-tangent.

6. Shew that impossible roots enter an equation by pairs.

7. A body falls 20 feet down an inclined plane, whose height is one-sixth of its length. What is the time of its motion, and what its last acquired velocity?

8. A body is projected at an angle of 45° with a velocity of 50 feet per second. Find its horizontal range.

9. A body weighs 14 lbs. in vacuo, and 9 lbs. in water. Another weighs 8 lbs. in vacuo, and 7lbs. in water. Compare their specific gravities.

10. A person who can see distinctly at the distance of 3 feet, wishes to see an object at 12 feet distance. What sort of glass must he use, and what must be its focal length.

11. Divide 30 into two such parts, that the square of the one multiplied into the other, may be a maximum.

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12. Find fluents of $\overline{a^2+x^2}|^{\frac{3}{2}} \times x\dot{x}$, of $\frac{x\dot{x}}{a^2+x^2}$, and of $\frac{a\dot{x}}{\sqrt{2ax-x^2}}$. Find also the fluxion of $\frac{y}{\sqrt{1+y^2}}$.

13. Sum 50 terms of the arithmetical series $1+3+5+$ &c. Also $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+$ &c. ad infin.

14. If a body revolves in a circle, how far must it fall externally and internally, to require the velocity in the circle: the force varying inversely as $\overline{\text{dist.}}|^2$?

15. Prove that in any curve, the centrifugal force: centripetal force :: $Sy^2 \times Pv : 2Sp^3$.

16. If the force varies as the distance, shew that the velocity of a falling body is as the sine, and the time as the arc; the versed sine being the space described.

17. Find the diameter of curvature in an ellipse, and the chords of curvature passing through the focus and through the centre.

18. Define the following terms in Astronomy: The right ascension and declination, latitude and longitude of an heavenly body; the true anomaly and mean anomaly of a planet; diurnal parallax and annual parallax; solstitial colure, and equinoxial colure.

Evening Problems.—Mr. Brown.

1. What is the present worth of an annuity of £20. a year, to continue for ever, and to commence after 2 years; the rate of interest being 5 per cent.?

2. How far will the Tychonic system account for the phenomena of the heavenly bodies? What phenomena will it not account for?

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3. A pendulum in the latitude of 60° oscillates once in a second, supposing the earth not to revolve round its axis. What must be the angular motion of the earth, that it may oscillate once in 3 seconds?

4. Compare the mean ablatitious force in a whole revolution of the moon with the force of gravity on the earth's surface.

5. Prove, in the latitude of Cambridge, that the diurnal path of a shadow formed on the horizontal plane by the vertex of a gnomon, is convex towards the north during the passage of the sun from the vernal to the autumnal equinox, and convex towards the south during his passage from the autumnal to the vernal equinox.

6. Find the probability of throwing two aces in three trials with a single die.

7. Shew in what position of the eye and object the image formed by a double convex lens will be magnified or diminished; and when it will be erect, and when inverted.

8. Construct the fluent of $\frac{\dot{z}}{1+2az+z^2}$, a being less than 1.

9. Find the time in which a body will describe a given space, when acted upon by a force varying inversely as the distance.

10. Find in MERCATOR'S projection the length of the meridian intercepted between 30° and 60° of latitude.

11. Find the fluents of $\frac{\dot{x}}{\overline{x-a}^2 \times \overline{x-b}}$; and of $\frac{(a+\sqrt{a^2-x^2})\dot{x}}{x}$; of $X^2\dot{x}$ where X=hyp. log. of x.

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Also of $\frac{\dot{x}}{x^4-2x^3-x^2+2x}$; the roots of the equation $x^4-2x^3-x^2+2x=0$ being -1, 0, 1, 2.

12. The radii of a wheel and axle are in the proportion of $a:b$; a weight P, acting by means of a line on the circumference of the wheel, elevates another weight 2, suspended from the line which goes round the axle. Required the pressure on the axis.

13. At a given place, and a given time, find the angle between the ecliptic and horizon, the culminating point, and the height of the nonagesimal degree. When is the angle between the ecliptic and horizon the least?

14. Find the expression for $\overline{\cos.x}^n$ in terms of the cosines of multiples of x.

15. In a given parabola to inscribe another parabola, whose area shall be a maximum; the vertex of the inscribed parabola being in the middle of the base of the other.

16. A given piece of gold is balanced by its weight of brass in vacuo. What addition must be made to the brass, that they may be in equilibrio when immersed in water?

17. Apply TAYLOR'S Theorem to find the sine of an arc in terms of the arc itself.

18. Sum the following series:

$1-\frac{1}{3.3.}+\frac{1}{5.3^2}-\frac{1}{7.3^3}+$ &c. ad. inf.

$\frac{1}{1.2.5}-\frac{1}{2.3.7}+\frac{1}{3.4.9}-$ &c. ad inf.

$1^4+2^4+3^4+$ &c. to n terms, by increments.

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19. Given the lengths of two degrees in different latitudes; Find the ratio of the polar and equatorial diameters.

20. The sum of the m^{th} powers of the roots of the equation $x^n - 1 = 0$ is n, whenever m is equal to n or to any multiple of n; but in all other cases $= 0$. Required a proof.

21. An iron globe descends in water by the force of gravity; Find the greatest velocity it can acquire, and the time of describing a given space from rest.

22. A stone is whirled round horizontally by a string 2 yards long. What is the time of one revolution, when the tension of the string is 4 times the weight of the stone.

23. Transform the equation $x^n - px^{n-1} + qx^{n-2}$ &c. $= 0$, whose roots are α, β, γ, &c., into one whose roots are $\overline{\alpha-\beta}^2$, $\overline{\alpha-\gamma}^2$, $\overline{\beta-\gamma}^2$, &c.; and shew the use of such a transformation in finding impossible roots.

Tuesday Morning Problems.—Mr. Brown.

1. Find the value of the decimal .346565, &c. in a vulgar fraction.

2. Shew how the latitude and longitude of a star may be found.

3. Two equal weights, P and Q, are connected by a string passing over a fixed pulley; What weight added to P will cause it to acquire in 6 seconds a velocity of 48 feet? and through what space will it have fallen in that time?

4. If $A + B + C = m\pi$, (m = any whole number, $\pi = 180°$); then, tan. A + tan. B + tan. C = tan. $A \times$ tan. $B \times$ tan. C.

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5. Find the whole fluent of $\frac{x^m \dot{x}}{\sqrt{a^2 - x^2}}$, when $x = 0$, and m is an even number. Find the fluents of $\frac{\dot{z}}{\sin. z}$, and of $\frac{y^{\frac{1}{2}} \dot{y}}{\sqrt{r^3 - y^3}}$.

6. Find the ratio between the sines of incidence and refraction out of a certain fluid into air; so that the eye, which can just see the extremity of the base of a given cylindrical vessel when empty, may see the centre of the base when it is three parts full.

7. Determine the ratio of the diameter and altitude of a cylindrical vessel open at the top, which shall contain a given quantity under the least possible internal superficies.

8. Find the expression for the proportion between the centripetal and centrifugal forces in any curve; and apply it to the reciprocal spiral.

9. Find the sum of the first n terms of $\frac{1}{1.2.4} + \frac{1}{2.3.5} + \frac{1}{3.4.6}$, by increments; and of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2}$, &c. ad inf.

10. Find the increment of the moon's velocity by the tangential force, while she moves from quadrature to syzygy.

11. Shew that, if the resistance in any medium be proportional to the velocity, the oscillations in a cycloid will be isochronous.

12. If a be root of the equation, $x^n - px^{n-1} + qx^{n-2} + \&c. \ldots\ldots \pm w = 0$; shew that the equation is divisible by $x - a$, without considering it as composed of the factors $\overline{x - a} . \overline{x - b}$. &c.

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13. Suppose a cylindrical rod, whose length *(l)* and weight *(w)* are given, to be suspended at a distance *(d)* from one of its ends. If a weight *(a)* be attached to the lower end, what weight must be attached to the upper, that the time of oscillation may be the same as before?

Third and Fourth Classes.—Mr. D'Oyly.

1. Find two values of x in the equation $x+\frac{24}{x-1}=3x-4$.

2. Given the three sides of a plane triangle; Explain how the three angles may be found.

3. Divide the number 100 into two such parts, that the square of the one multiplied into the square root of the other, may be a maximum.

4. Find between which of the roots of the equation $x^3-7x^2+7x+10=0$, the number 3 lies.

5. A stone projected at an angle of 30° strikes the horizontal plane at the distance of 50 yards. With what velocity was it projected?

6. A pendulum vibrates 61 times in a minute. How must its length be altered, so that it may vibrate 60 times in a minute?

7. In the tube of a barometer 36 inches long, the mercury stands at 29¼ inches, when the standard altitude is 30 inches. How much air was left in at the time of inversion?

8. Find the principal focus of a given glass globe.

9. Find the area of the curve, in which the subtangent $=\frac{b}{1+y^2}$; y being the ordinate, and b a constant quantity.

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10. Find fluents of $\frac{\dot{x}^3\dot{x}}{x-a}$, of $y^3\dot{y}\sqrt{a^2-y^2}$, and of $a^x x\dot{x}$.

11. Explain the method of finding the hour of the the day by observing the sun's altitude; the latitude of the place, and the sun's declination, being known.

12. If a hole were bored from the surface of the earth to its centre, what would be the velocity acquired at the centre, by a body which fell from the height of 1 radius above its surface; the force above the surface being inversely as $\overline{\text{dist.}}^2$, and below the surface being directly as dist.?

13. Having given the major and minor axis of an ellipse, and the force in the focus; Compare the periodic time in the ellipse with the periodic time in a circle, whose radius = greatest distance in the ellipse.

Fifth and Sixth Classes.—Mr. Brown.

1. Find the value of the decimal .124343 &c. in a vulgar fraction.

2. If $\frac{1}{4}$ of a yard of cloth cost $\frac{1}{4}$£., what will $12\frac{1}{2}$ yards cost?

3. At what time are the hour and minute-hands of a watch together between 2 and 3 o'clock?

4. Find the length of the cycloid.

5. Given $\left\{\begin{matrix} x^3+y^3=72 \\ x+y=6 \end{matrix}\right\}$ to find x and y.

6. Given the latitude of the place, and the sun's meridian altitude; to find the sun's declination.

7. Shew, that the velocity in a parabola is to the velocity in a circle at the same distance as $\sqrt{2}:1$.

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8. Sum the series:

$$\frac{1}{3}-\frac{1}{6}+\frac{1}{12}-\frac{1}{24}+\&c.\text{ ad inf.}$$

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\&c.\text{ ad inf.}$$

9. Two columns of water and mercury communicate at the bottom of a bended tube. If the altitude of the mercury be 10 inches, what is the altitude of the water, supposing the ratio of their specific gravities to be 14 : 1?

10. What is the length of the longest day in latitude 45°?

11. Given the tangents of two arcs, to find the tangents of their sum and difference.

12. When the force varies inversely as the square of the distance, compare the velocity acquired through any space with the velocity in a circle at the same distance.

14. Shew, that the velocity in any curve, is equal to that which would be acquired with the same force continued uniform, through $\frac{1}{4}$ chord of curvature.

13. Find the field of view, and magnifying power, of the Astronomical telescope.

14. Find the fluxion of $\frac{\overline{a^4+x^4}^{\frac{1}{2}}}{x}$, and the fluent of $\frac{2a\dot{x}}{a^2+x^2}$.

16. Inscribe the greatest parallelogram in a given semi-circle.

Evening Problems.—Mr. D'Oyly.

1. Find the hyperbolic logarithm of 3, by applying COTES'S series for the hyp. log. $\frac{z+x}{z-x}$ to 3 terms.

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2. Find an arc, which has the rectangle under its chord and cosine a maximum.

3. Approximate to a value of x in $x^3+2x-30=0$.

4. An object placed 4 inches before a double convex lens, has its image erect with respect to itself, and of three times its linear magnitude.—Required the focal length of the lens.

5. A body is projected at an angle of 60° to the horizon, with a velocity of 100 feet per second. What will be its time of flight before it strikes a plane inclined to the horizon at an angle of 30°? and what will be its greatest elevation above that plane?

6. Investigate Sir I. NEWTON'S rule for detecting impossible roots in an equation, and apply it to the equation $x^6+3x^4-4x^2-12=0$.

7. If a quadrantal arc is divided into any 3 parts, the sum of the products of the tangents of these parts taken two together, is equal to the square of the radius. Required proof.

8. The right ascensions and declinations of two stars being known, the time is observed when they come to the same azimuth. Shew how the latitude of the place may from hence be found.

9. Prove that the portion of the earth's surface included between the two circles of latitude 30° on each side of the equator, is half its whole surface; the earth being supposed to be a perfect sphere.

10. Find fluents of $\frac{x^6\dot{x}}{x^2+a^2}$ of $\frac{\dot{x}}{x^5\sqrt{x^2+a^2}}$, and of $\frac{\dot{x}}{\sin. z \cos. z}$.

11. A chain of uniform thickness, hanging freely, forms itself into a cubical parabola. Required the law of its density and weight.

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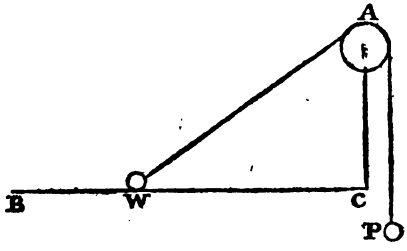
12. Let a plane circle oscillate on an axis, which passes perpendicular to its plane at the distance of half the radius from its centre. At what other distance may the axis pass, so that the oscillation may be performed in the same time?

13. If a body revolve in an ellipse round the focus, required the point at which the angular velocity of the perpendicular on the tangent is the least possible.

14. If a body is revolving in a circle $\left(F \propto \frac{1}{D^3}\right)$, and half the central force is suddenly taken away; after what revolution will it fly off to an infinite distance?

15. The node of the moon's orbit being distant 30° from syzygy, the part of the ablatitious force, which acts perpendicular to the plane of the orbit, is found to be $\frac{1}{4}$th of its greatest value. What is the moon's place?

16. Let P and W be two bodies connected by a string passing over the pulley A; P hanging freely,



and W being sustained on an horizontal plane. What velocity will be acquired by W, while it is drawn by P through the given space BC.

17. Integrate the fluxional equation $a\dot{x} = \frac{x\dot{y}\ddot{y} + x\dot{y}^2}{\dot{x}}$.

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18. A given parabola is placed with its plane and axis perpendicular to the horizon, and its vertex downwards. From what point of its highest ordinate must an elastic body be let fall, so that after impinging once on the curve, it may strike the vertex?

19. Find the increment of $z \times \log. z$.

20. Let a system of bodies be moveable round a vertical axis. At what distance from this axis must a given force act, so that the angular velocity communicated to the system in a given time, may be the greatest possible?

21. A person is throwing with two dice. What is the chance of his throwing size-ace, at least once in four trials?

22. Sum n terms of the series $1.2.5 + 2.3.6 + 3.4.7 + \&c.$

Also n terms of $\frac{2}{1.3.4.} + \frac{3}{2.4.5.} + \frac{4}{3.5.6} + \&c.$

And $\frac{1}{1.5} - \frac{1}{3.7} + \frac{1}{5.9} - \frac{1}{7.11} + \&c.$ ad infin.

23. Given the place of a luminary, and a situation on the earth's surface; Find how much the superior tide exceeds the inferior in duration.

24. A beam of given length, having its perpendicular section every where a given parabola, projects horizontally from a wall. Compare its strength to support a weight at its end, when the vertex of the parabola is downwards, with its strength, when the vertex is upwards; the weight of the beam itself not being considered.

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1810.

First Morning.—Mr. Walter.

MONDAY, JANUARY 15.—*FIRST AND SECOND CLASSES.*

1. Prove the rule for the multiplication of decimals.

2. What number of degrees, minutes, &c. in the English scale corresponds to 32°. 15′. 25″. in the French scale of 400° in one circumference, 100 minutes in a degree, and 100 seconds in one minute?

3. The quantities of air expelled, by successive turns, from the receiver of an air-pump, decrease in geometrical progression.

4. Given the distance of a planet from each of two stars, whose right ascension and declination are known. Find the right ascension and declination of the planet.

5. If any odd multiple of a quadrant be divided into any three parts, the sum of the co-tangents of the three parts will equal their product.

6. If from a given point a straight line be drawn touching a circle given in position; the straight line is given in position and magnitude.

7. Prove that the sub-contrary section of an oblique cone is a circle.

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8. On the axis of a given cycloid, ordinates are drawn equal to the corresponding arcs of the cycloid: find the nature of the curve passing through the extremities of these ordinates, and its area.

9. The aspect of a wall is due south, and the sun is in the south-east with an altitude of 30°. It is required to find the breadth of the shadow cast by the wall.

10. Find the sum of the series $\frac{a}{c}+\frac{a+b}{ce}+\frac{a+2b}{ce^2}+\frac{a+3b}{ce^3}+$&c. ad inf.

11. Given the capacity of the recess containing compressed air, the space which that air would occupy in its natural state, the diameter and weight of the ball, and the length of the barrel of an air-gun: find the velocity with which the ball leaves the gun, and the time of describing the barrel.

12. If a body be projected from an apse with the velocity acquired in falling from an infinite distance, find after how many revolutions it will fall to the centre when the force varies in a higher ratio than the inverse cube of the distance.

13. Shew that the series $\sin. A+\sin.(A+B)+\sin.(A+2B)+$&c. is a recurring series; and find the scale of relation.

14. A body being projected along any curve, and acted on by gravity, it is required to find an equation for determining the point where it will leave the curve.

Third and Fourth Classes.—Mr. Turton.

1. If $1\frac{1}{8}$th ell English cost $\frac{5}{6}$ths of a guinea, what will $2\frac{1}{3}$d ells Flemish cost; a Flemish ell being $\frac{3}{5}$ths of an English ell.

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2. If an angle of a triangle $=120°$, shew that the square of the side subtending that angle is equal to the squares of the sides containing it, together with the rectangle contained by those sides.

3. Of two rays passing through a spherical refracting medium, shew that the deviation of the more remote from the centre is greater than that of the less remote.

4. Find at what distance from a given sphere an eye must be placed so as to see an nth part of its surface.

5. The plane of a cycloid, whose axis $=a$, is inclined to the horizon at an angle of 60°. Find the time of descent down a chord drawn from the vertex to the extremity of the base.

6. Prove that in all the conic sections the chord of curvature through the focus $=L\times\frac{SP^2}{SY^2}$, and the diameter of curvature $=L\times\frac{SP^3}{SY^3}$; where L is the principal latus rectum, SP the focal distance, and SY the perpendicular on the tangent.

7. If the force vary directly as the distance from the centre, shew that the velocity of a falling body is to that in a circle at the same distance as the right sine to the cosine of a circular arc, whose radius is the greatest distance and versed sine the space fallen through.

8. Construct Galileo's Telescope; investigate its magnifying power, and find the linear magnitude of the greatest field of view.

9. Find the fluent of $\frac{dx^{3n-1}\dot{x}}{\sqrt{a^n-x^n}}$, and of $\frac{dx^{3n-1}\dot{x}}{a^n-x^n}$.

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10. Find the roots of the equation $x^4+3x^3-7x^2-27x-18=0$, two of which are of the form $+a, -a$.

11. If a body fall through a finite altitude AS, the force varying inversely as the square of the distance, and on AS a semi-circle ADS be described; prove that the area described by the variable radius SD is equal to the area uniformly described in the same time in a circle whose radius is the half of SA.

12. Through what space must a body fall internally to acquire the velocity in a circle, the force varying inversely as the distance.

13. Given the sun's meridian altitude, and his midnight depression below the horizon: find the latitude of the place, and the sun's declination.

14. Given the major and minor axis of an ellipse, and the force in the focus. Compare the time in which a body would fall from the farther apside to the focus with the periodic time in the ellipse.

15. Prove that, in different circles, round different centres, when the forces vary inversely as the squares of the distances, the periodic times will vary in the sesquiplicate ratio of the radii directly, and the subduplicate ratio of the absolute forces inversely.

16. If the forces of attraction to each point of a spherical superficies vary inversely as the square of the distance, the attraction of the convex and concave part, on a *corpuscle* placed without it, will be equal.

Fifth and Sixth Classes.—Mr. Walter.

1. Required the value of 2710 lbs. at 6s. 8d. per lb.

2. Required the interest of £547. 15s. at 3 per cent. simple interest, for three years.

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3. A certain number, consisting of two figures, gives, when divided by the figure on the right, a quotient equal to 27, and remainder 2; but, if divided by 9, it gives a quotient equal to three times the figure on the right, and a remainder 2. Find the number.

4. Find the value of x in $\sqrt{x}-4=\frac{259-10x}{4+\sqrt{x}}$.

5. Prove, that if $A \propto B$, and $C \propto D$; then $AC \propto BD$.

6. Compare P and W, when they are in equilibrio on a plane whose inclination is 30°, the power being supposed to act parallel to the base of the plane.

7. One body remains at rest in the circumference of a circle, whilst another describes that circumference. It is required to ascertain the curve described by their common centre of gravity.

8. Bisect a given semi-cycloidal arc.

9. Prove, that the horizontal range of a body, projected in the direction AI, is equal to $\frac{\text{sin. } 2 \angle IAC \times P}{2 \text{ . rad}}$; and prove sin. $2. \angle IAC =$ sin. $2 \angle EAC$.

10. Solve the equation $x^3-5x^2-8x+48=0$; two of whose roots are equal.

11. Sum the series $1+\frac{1}{3}+\frac{1}{9}+$ &c. ad inf.;

and $1-2+4-8+$ &c. to n terms.

12. Compare the pressure on the base of a cube, filled with water, with the pressure on the four sides.

13. $2C : qC ::$ sin. $R :$ sin. I, when diverging rays are incident on a plane refracting surface.

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14. Given the altitude of a known star, when it is on the prime vertical; find the latitude of the place.

15. Find the fluxion of $\frac{x}{\sqrt{a^3+x^3}}$; also the fluent of $\frac{a\dot{y}}{\sqrt{a^2-y^2}}$.

16. Prove, that the velocity of the earth continually increases from the aphelion to the perihelion distance.

17. Compare the velocity in an ellipse, at the greatest distance, with the velocity of a body revolving in a circle at the same distance.

Evening Problems.—Mr. Turton.

1. Explain the nature of the stereographic projection of the sphere: and shew that angles on the plane of projection are equal to the original angles on the surface of the sphere.

2. Prove, by means of Newton's fourth Lemma, that the area of the common parabola is equal to two-thirds of its circumscribing rectangle.

3. Having given a rectangular piece of metal of uniform thickness, it is required, from one of its angles, to draw a line cutting off a triangle, in such a manner, that, the remaining trapezium being suspended by the obtuse angle, the parallel sides of the trapezium may remain horizontal.

4. It is observed, that two stars, whose right ascensions and declinations are known, pass the prime vertical at the same instant. Required the latitude of the place of observation.

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5. Given the latitude of the place, and the sun's declination: find at what time of the day the azimuth of the sun increases the slowest.

6. Investigate, as Cotes has done, the variation of the density of the atmosphere, supposing the force of gravity to vary inversely as the square of the distance from the earth's centre.

7. One root of the equation $x^3 - 4x^2 - 3x + 12 = 0$, is of the form $\sqrt{a}$, where a is not a square number. Solve the equation.

8. A given paraboloid is perforated by a cylinder whose axis coincides with that of the solid. Required the dimensions of the cylinder, so that the part taken away may be equal to that which remains.

9. In a combination of wheels and axles, in which the circumference of each axle is applied to the circumference of the next wheel, and in which the ratios of the radii of the wheels and axles are those of 2 : 1, 4 : 1, 8 : 1, &c. there is an equilibrium when the power : the weight :: 1 : p. Required the the number of wheels.

10. If the roots of the equation $x^n - px^{n-1} + qx^{n-2} - \&c. = 0$, be in arithmetic progression, the least root will be $\frac{p}{n} - \frac{n-1}{n} \times \sqrt{\frac{n-1 \,.\, 3p^2 - 6nq}{n^2 - 1}}$, and the common difference $\frac{2}{n} \times \sqrt{\frac{n-1 \,.\, 3p^2 - 6nq}{n^2 - 1}}$. Required the investigation.

11. Given the major and minor axes of an ellipse. Required the radius of a circle described round the focus as a centre, in which the periodic time is equal to the time of moving through the aphelion, from mean distance to mean distance.

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12. Construct Newton's telescope, and investigate its magnifying power.

13. Compare the quantity of water discharged by two equal parabolas in the side of a reservoir kept constantly full; one of the parabolas having its base, and the other its vertex downwards, and the summits of both coinciding with the surface of the fluid.

14. Suppose that a body falls from a given altitude to a centre of which the attractive force varies as the distance, and that the system moves in a direction perpendicular to the line of descent, with a velocity equal to the greatest velocity which the body could acquire in its fall; construct the curve traced out by the body, and then find its area.

15. If z be an integral increasing or decreasing unequally; then $\frac{1}{z}_{m} = \frac{1}{z} - \frac{\dot{z}}{z\dot{z}}_{1} - \frac{\dot{z}+\ddot{z}}{z\dot{z}}_{1\,2} - \frac{\dot{z}+2\ddot{z}+\dddot{z}}{z\dot{z}}_{2\,3}$ - &c. continued to $m+1$ terms.

16. Find the fluents of the following quantities: viz.

$\frac{d\dot{x}}{x^3\sqrt{a^2+x^2}}$; $\frac{x^2\dot{x}}{(x-a)(x-b)(x-c)}$; $\overline{a^2+x^2}^{m} \times \dot{x}$, when the fluent of $\overline{a^2+x^2}^{m+r} \times \dot{x} = A$; $z^n\dot{y}$, where z is a circular arc, and y its sine to radius 1.

17. Sum the following series:

$\frac{1}{1.4.7} + \frac{1}{4.7.10} + \frac{1}{7.10.13}$ + &c. ad infin.

$\frac{}{2^2} + \frac{}{5^2} + \frac{}{8^2}$ + &c. to n terms.

$\frac{1}{1.2} + \frac{1}{5.6} + \frac{1}{9.10}$ + &c. ad infin.

$\frac{2}{1.3.3} + \frac{3}{3.5.3^2} + \frac{4}{5.7.3^3}$ + &c. to n terms.

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18. Materials are to be raised, through a given altitude, by a wheel and axle whose radii are known; the power, which is given, being applied to the circumference of the wheel. Find the quantity raised at each ascent when the greatest quantity in the whole is raised in a given time; the inertia of the machine being neglected.

19. Find the area on the plane of the horizon that is bounded by the shadow of a tower of given altitude, between the hours of 8 and 2, in a given latitude: the sun being in the equinoctial.

20. Find the relation of x to y in the equation $\dot{x}(a+bx+cy)=\dot{y}(d+ex+fy)$.

21. Let a sphere of given diameter be projected in a fluid whose specific gravity is to that of the body as 1 to n: Having given the velocity of projection, it is required to find the velocity after describing any space and also the time of describing it.

22. Prove that the projection of the rhumb-line, on the plane of the equinoctial to an eye situated in the pole, is a logarithmic spiral; and hence determine the length of any arc of the meridian, on the planisphere.

23. Determine, as Newton has done, the path of a projectile in a medium in which the resistance varies as the velocity; the force of gravity being uniform and acting in parallel lines.

24. Determine the dimensions of a conic frustum, of given altitude, on which, when moving in a resisting medium, in the direction of its axis, with its less end foremost, the resistance will be equal to that on the base of a given cylinder, moving with the same velocity; and at the same time, less than the

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resistance on any other frustum of the same base and altitude.

Morning Problems.—Mr. Turton.

1. Define similar curves when referred to their axes; and prove that similar and conterminous arcs have a common tangent at the common point of termination.

2. If *(a)* and *(b)* be two sides of a trapezium that are parallel to each other; prove that the centre of gravity of the figure will divide a perpendicular to those sides into two parts that are to each other as $2a+b$ to $2b+a$.

3. If a body fall from a finite altitude towards a centre of force, and the time of falling vary as the nth power of the space fallen through; required the law of the variation of the force.

4. Resolve $\frac{1}{1-z^n}$ into trinomial fractions without the aid of fluxions; n being an even number.

5. If the refracting curve be the logarithmic spiral, and rays issue from the centre, investigate the nature of the caustic.

6. Find the force of elasticity, so that, in the case of direct impact, the sum of the products of each body into the *cube* of its velocity may be the same before and after impact.

7. If a weight P be suspended by an inflexible line, whose length is (a), to what point must a given weight p be attached, so that the pendulum may oscillate in the least time possible?

8. There is a small aperture, whose area is (m) at a given distance (a) from the bottom of a vertical

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cylinder filled with water. When full, the fluid falls on the horizontal plane, at the distance (b) from the base; and after (t) seconds at the distance (c) from the base. Required the content of the vessel.

9. Let $y = A + Bx^m + Cx^n + Dx^p$ + &c. where A, B, C, &c. are constant quantities: then if $\dot{x}$ be given, $y = \frac{\dot{y}}{\dot{x}} \times \dot{x} + \frac{\ddot{y}}{1.2.\dot{x}^2} \times \dot{x}^2 + \frac{\dot{y}}{1.2.3.\dot{x}^3} \times \dot{x}^3$ + &c. Required the investigation.

10. Find the fluent of $\frac{d\dot{z}^{pn+\frac{1}{2}n-1}\dot{z}}{a+bz^n}$; and also of $\frac{dz^{\frac{1}{2}n-1}\dot{z}}{\sqrt{a+bz^n}}$.

11. Find the amount of £1. for the time (t), at compound interest at a given rate; interest being due every moment.

12. In a given latitude, at a given hour, and on a given day, the altitude and azimuth of a star are observed. Required its right ascension and declination.

13. Suppose the earth a perfect sphere, and that a pendulum whose length is (a) inches, vibrates seconds in latitude 60°. What will be the length of a pendulum that vibrates seconds at the equator?

14. Deduce Cotes's construction of his first spiral, by means of Newton's general proposition in the 8th section.

Tuesday Afternoon.—Mr. Walter.

THIRD AND FOURTH CLASSES.

1. How much ready money can I receive for a note of £75. due 15 months hence, at 5 per cent. discount.

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2. Find the values of x and y, when $xy=63$, and $\overline{x+y}|^2 :: \overline{x-y}|^2 : 64 : 1$. Also value of x in this equation $x^2.\overline{x+4}+2x.\overline{x+4}=2-\overline{x+4}$.

3. Divide a given angle into two angles, such that their sines may be in a given ratio.

4. Express the side of a regular decagon, inscribed in a circle, in terms of the radius.

5. Two bodies, A and B, are projected perpendicularly upward with velocities (a) and (b). It is required to assign the highest point to which their common centre of gravity will ascend.

6. Find the roots of the equation $x^3-13x^2+50x-56=0$, two of whose roots are in the ratio of 2 : 1.

7. The diameter of a cylinder is 10 inches, and the diameter of an orifice in its base .025; also the height of the water in the cylinder is $8\frac{1}{24}$ feet. Required the time of emptying.

8. Given the apparent perpendicular depth of an object under the water, to find the direction in which a ball must be fired from a given point, so as to strike the object.

9. Sum the series $3+\frac{1}{2}+\frac{1}{12}+$&c. *ad infinitum*. Also, $1.3.5+3.5.7+5.7.9+$&c. to n terms.

10. Determine the equation of a curve by whose revolution a solid is generated equal, at all altitudes, to $\frac{1}{3}$ths of its circumscribing cylinder.

11. Find the centre of gravity of a bar whose density $\propto x^n$; x being the distance from the vertex.

12. A known star rises in the north-east point; find from this circumstance the latitude of the place.

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13. Prove that V^2 in any curve : v^2 in a circle at the same distance :: $\frac{\dot{y}}{y} : \frac{\dot{p}}{p}$, where y is the variable distance, and p the perpendicular on the tangent.

14. How does the centrifugal force vary in different curves? and how does it vary in different parts of the same curve?

Fifth and Sixth Classes.—Mr. Turton.

1. If $\frac{3}{5}$ yards cost £2$\frac{1}{4}$, find the value of 5$\frac{7}{8}$ yards both by vulgar fractions and by decimals.

2. Prove that if a straight line stand at right angles to each of two straight lines in the point of their intersection, it will be at right angles to the plane that passes through them.

3. Define a rhombus; and prove that the diagonals of a rhombus bisect each other at right angles.

4. If (a) be the first term, and (b) the sum of three terms of a geometric progression, find the common ratio.

5. If the fluxion of $\frac{\sqrt{ax} - b}{\sqrt{a - x}} = 0$, find the value of x.

6. Given three bodies A, B, C, and their distances from a plane; find the distance of their common centre of gravity from that plane, supposing A and B to be on one side of the plane, and C on the other side.

7. Given the velocity and direction of projection, find the greatest height of the projectile above the inclined plane; and from the expression deduce the greatest height above the horizontal plane.

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8. Shew that if a plane mirror recede from a fixed object, the image will recede twice as fast.

9. Explain the principle on which the Hydrometer is constructed, and demonstrate the proposition on which the construction depends.

10. Construct the common Astronomical Telescope, and investigate its magnifying power.

11. Given the latitude of the place, and the sun's declination; to find his azimuth at six o'clock.

12. Shew that the velocity in any conic section is to the velocity in a circle at the same distance in the subduplicate ratio of $\frac{1}{2}L \times SP$ to SY^2.

12. Find the fluents of the following quantities: viz. $\frac{\dot{z}}{\overline{a+bz}|^{\frac{m}{n}}}$; $\frac{x^5\dot{x}}{a^2-x^2}$; and $\overline{a^2+x^2}|^m \times x^2\dot{x}$.

14. Sum the series $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5}$ + &c. to n terms, and *ad infinitum*.

15. Investigate the assumptions by which an equation $(x^n - px^{n-1} + qx^{n-2} - \&c. = 0)$ may be transformed into others wanting the second or third terms.

16. Given the earth's radius and the space fallen through in one second at its surface, find the periodic time in a circle at a given distance above the earth's surface; gravity varying inversely as the square of the distance.

17. If a body whose elasticity is to perfect elasticity as m to 1 be let fall from a given altitude (a) above a perfectly hard horizontal plane, and rebound continually till its whole velocity is destroyed; find the whole space described.

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18. A given paraboloid floats in a fluid with its vertex downwards; compare the specific gravities of the body and the fluid, supposing half the axis to be immersed.

Evening Problems.—Mr. Walter.

1. A cylindrical bar is suspended by a given point in a semi-circle, whose diameter is the bar. Find the inclination of the bar to the horizon, upon supsition that the semi-circle is devoid of weight.

2. Prove, from a property of the circle, that if four quantities are proportionals, the sum of the greatest and least is greater than the sum of the other two.

3. Given the area of any plane surface, it is required to find the content of a solid, formed by drawing lines from a given point without the plane, to every part of its surface.

4. The inclination of a perfectly smooth bank to the horizon is 30°, and a body is projected up the bank in a direction making an angle of 45°, with the intersection of the bank and horizontal plane. It is required to determine the curve described by the body, and the spot where it will again meet the horizon.

5. If two curves have a common axis, and ordinates which are always in a given ratio to each other, then tangents drawn from the extremities of any corresponding ordinates will meet the axis in the same point.

6. The direction of a bridge is from east to west, and the sun in the meridian. The arches being supposed semi-circular, it is required to find the curve

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terminating that part of the surface of the water which is illuminated by the sun's rays passing through any arch.

7. It is required to express the cosine of an angle of a spherical triangle in terms of the sines and cosines of the sides.

8. If a body revolves in any curve whose equation is $ap=y^n$, y being the distance from the centre of force, and y the perpendicular on the tangent; it is required to find the equation of the curve of a star's apparent aberration, as seen from this body.

9. The roots of the equation $x^3-px^2+qx-r=0$, are a, b, c,; transform it into one whose roots shall be $\overline{a+b}$, $\overline{b+c}$, $\overline{a+c}$.

10. Required the position of the eye in a given line perpendicular to the horizon, so that the image of a given circle on the ground may also be a circle, when projected on a plane perpendicular to the horizon by lines drawn to the eye.

11. Find by the help of the common tables the logarithm of a number consisting of seven figures.

12. The roots of the equation $x^3-px^2+qx-r=0$, are in harmonical progression: find them.

13. Given the sun's declination, and the latitude of the place; find the path described by the shadow of a staff on an horizontal plane.

14. Sum the series $1.2.4+3.4.6+5.6.8+$&c. to n terms.

Also $\frac{1}{3}-\frac{2}{4}+\frac{3}{5}-\frac{4}{6}$&c. to infinity.

And $\frac{1}{1.3}+\frac{1}{2.4}+\frac{1}{3.5}+$&c. to n terms, and to infinity.

15. A has (p) counters, and B has (q); also the

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chance of *A's* winning a single counter from *B* is to the chance of *B's* getting one from *A* :: $a : b$. What is the probability that *A* will win all *B's* counters?

16. Explain the method of finding the velocity with which a bullet struck a pendulous body.

17. If a ponderous cylinder is put in motion about its axis by a weight p, descending through a space s; and, p being taken off, the moment thus generated be employed to elevate another weight q: the space through which the cylinder's motion elevates q can never be so great as $s \times \frac{p}{q}$, even if the effects of friction are neglected.

18. Find the fluent of $v^m x^n \dot{x}$, when v is the hyp. log. of $\frac{1}{x^2}$. Also the fluent of $\frac{x^2\dot{x}}{x^2-3x+2}$.

18. Find the relation of x and y, when $\frac{p\dot{x}}{x} + \frac{r\dot{y}}{y} = \frac{x^m\dot{x}}{ay^n}$, and also when $ay + \frac{b\dot{y}}{\dot{x}} + \frac{c\ddot{y}}{\dot{x}^2} = 0$.

20. Having given the velocity of projection, greater than that acquired by falling from an infinite distance, when the force $\propto \frac{1}{D^3}$, and also the distance from the centre of force; find the proper direction of projection so that any one of Cotes's three last spirals may be described.

21. Having given the fluents of $y\dot{x}$, $yx\dot{x}$, $yx^2\dot{x}$, ... $yx^{n-1}\dot{x}$; find the fluents of $\dot{A} = y\dot{x}$, $\dot{B} = A\dot{x}$, $\dot{C} = B\dot{x}$ ..... $\dot{R} = Q\dot{x}$, the number of the last equations being n.

22. S is the pole of the logarithmic spiral, and P any point in the curve; if with centre P, and radius

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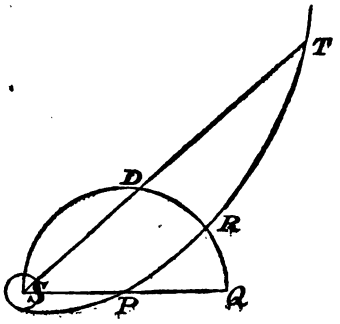
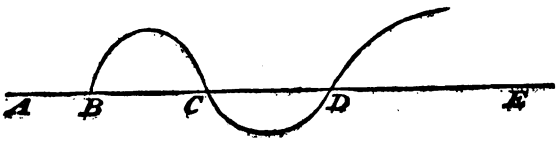
PS, a circle be described, and $2D$ taken equal to n times $2R$, then DT may be expressed by a series of cosines of $R2$ and its multiples, the variable coefficients of the series being those of $\overline{a+x}|^n$



23. Suppose both sun and moon in the equator, and prove that the momentary change in the height of the whole tide is proportional to the sine of twice the moon's distance from the place of high water.

24. Let $y = ax^n + bx^{n-1} + cx^{n-2} +$ &c. be the equation to the parabolic curve whose axis is AE. Prove that the continued product of the greatest ordinates will be equal to $\frac{a^{n-1}}{n^n}$ multiplied into $BC^2 \times BD^2 \times CD^2 \times$ &c.



FINIS.

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ERRATA.

P. 23. l. 18. for *a body*, read, *a falling body*.

P. 30. l. 4. read, the equation $a^2y - x^2y - a^3 = 0$.

P. 84. l. 17. for *vertices*, read *vortices*.

P. 108. l. 2. for *field view*, read *field of view*.

P. 113. l. 9. for *x cosec. p. sec. l.* read $\times$ *cosec. p. sec. l.*

P. 142. l. 11. for *cosines*, read *cosines*.

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